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ABSTRACT

This report on the 1974 testing of the Michigan Educational Assessment Program (MEAP) begins with a summary of the results of the 1973 tests and a discussion of changes made in the testing program. In 1974, thirty objectives were tested at grade 4 and forth at grade 7. For each of these objectives, this volume presents a sample item, a brief discussion of student performance on items testing the objective, remarks concerning the diagnosis of specific difficulties with the objective, and suggestions for classroom activities. Suggestions are offered to local education agencies for the use of assessment results in establishing priorities for curricular and instructional change. Specific advice for staff orientation and management strategies for teachers are also offered. Diagnostic tests are provided for classroom use. Appendices provide a summary of statewide assessment results and a detailed list of changes made in MEAP tests between 1973 and 1974. (SD)

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MICHIGAN EDUCATIONAL ASSESSMENT PROGRAM MATHEMATICS INTERPRETIVE REPORT

1974 Grade 4 and 7 Tests

Horace Mourer
MCTM

Leah M. Beardsley
Terrence G. Coburn
Alan A. Edwards
Joseph N. Payne

Monograph No. 8
September 1975

SE 020 687

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MICHIGAN EDUCATIONAL ASSESSMENT PROGRAM

MATHEMATICS INTERPRETIVE REPORT

1974 GRADE 4 AND 7 TESTS

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June 1975

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I

INTRODUCTION

The purpose of this monograph is to make interpretative remarks about the results of the 1974 mathematics assessment. The Grade 4 and Grade 7 benchmark tests were adminstered statewide in October, 1974. This is the second year of objective-referenced testing in Michigan. This report emphasizes specific teaching and curriculum suggestions. These suggestions contain diagnostic questions and stress appropriate teaching sequences, especially in areas where the levels of attainment appear to be weak.

The major message of this report is directed to the classroom teacher and is concerned with how to use the results of the mathematics assessment. The writers hope that individual teachers at all levels, K-7, will conduct further diagnostic testing in those areas where their students showed some weaknesses. Section IV consists of a discussion on how a local district can proceed in interpreting and using its own results. Section V contains several sample diagnostic tests.

The report does not attempt to make comparisons between the 1973 and 1974 results. The reader is referred to Monograph No. 7, January 1975 for a report on the 1973 results. At the statewide level, there is no information available on any of the possible factors to which differences in performance could be attributed. Some of the differences in performance may be attributed to changes in some of the test items. In general, the percents attaining each objective were, on the average, about 4% higher for the 1974 Grade 4 test. One dramatic exception to this pattern was Objective 29 which showed a performance level 21% lower than the 1973 results. Much of this loss can be attributed to a change in one test item measuring this objective. See Section III for details.

Alterations were made to the foils of many of the items on the Grade 7 tests. Most of these were related to fraction and decimal objectives. Almost without exception these changes were accompanied by lower results on the 1974 test. When one considers objectives other than those which were altered, the 1974 results were about 2% higher than the 1973 results.

II

PRESENTATION OF DATA

Five of the 35 objectives assessed in Grade 4 in 1973 were not tested in 1974. Some of the test items for the remaining 30 objectives were slightly modified. Five of the 45 objectives assessed in Grade 7 in 1973 were also deleted from the 1974 testing. See Appendix B for details on the changes made from 1973 to 1974 testing for both the Grade 4 and the Grade 7 tests.

Alterations to the 1974 tests were largely influenced by the MCTM's 1974 conference on State Assessment in Mathematics. Mathematics educators from all over Michigan convened for two days in April of 1974 and developed numerous specific recommendations directed at the objectives tested, the test items, the test procedures, and needed revisions to objectives in the book, Minimal Performance Objectives for Mathematics Education in Michigan (MPOMEM). Both the Michigan Educational Assessment Program (MEAP) staff and members of the General Education Services were very cooperative and have encouraged MCTM to continue to suggest improvements in the mathematics assessment. The 1975 conference was held in March at Central Michigan University. The interpretive remarks made by the writers in this monograph reflect the discussions and recommendations made at the conference. The report of the conference was presented to Michigan Department of Education staff in April. At this writing, it is too early to determine the action taken on the conference recommendations.

The major ingredients used in forming the interpretations and teaching suggestions presented herein included the following: interaction and insights gained at the March conference; data provided by MEAP consisting of the statewide percent of attainment for each objective (See Appendix A) and foil analysis (percent choosing each choice for each test item); and professional experience and judgment.

As in 1973, five multiple-choice test items were used for each objective. To be recorded as attaining an objective, a student must have responded correctly to at least 4 of the 5 test items on that objective. The percents of students who attained each objective are summarized in Tables 1 and 2. In the left column are the content areas. At the top are the ranges of percents of students attaining the objective. The numbers inside the tables identify the objective.

TABLE 1
1974 FOURTH GRADE TEST
PERCENT OF STUDENTS ATTAINING EACH OBJECTIVE

TOPIC	85-100	70-84	55-69	40-54	0-39
Pre-Number	1*2 4 5	3			
Numeration	9 10 12 14 17	6 7 8 11 13 15 18^b 19	16^b	20^c	
Addition	21				
Subtraction	25 26	[illegible] 24	22		
Measurement:					
Time	27				
Money		28			
Temperature				29	
Geometry		30			

*These numbers are the objective identification numbers for testing purposes. See Section III for full description.

b and c refer to special designations described on page 6. Objective numbers without designating letters are judged to be in category a.

TABLE 2

1974 SEVENTH GRADE TEST

PERCENT OF STUDENTS ATTAINING EACH OBJECTIVE

TOPIC	85-100	70-84	55-69	40-54	0-39
Numeration	2	1			
Addition	3 4				
Subtraction		5			
Multiplication	6 7 8 9				
Division	10	11^b 12	13		
Fractions:					
Meaning		15	14	16	
Addition			17 18		
Subtraction			19	20	
Multiplication			21		
Decimals:					
Meaning	22				23
Add/Subt		24			25 26
Ratio				$27^{b,c}$	
Measurement:					
Area				28	
Volume					$29^{b,c}$
Time		31	30		
Money		32		33	
Temperature		34			
Geometry		36	35		
Algebra	39	37	38	40^c	

The interpretation categories used by the writers of this report are as follows:

a. Minimal and appropriate. The objective is judged to be minimal and the test items appear to be appropriate. If the level of attainment is not at or above 85%, it may be due to inappropriate instructional materials or lack of proper emphasis in years prior to the test year. Objectives in this category are unlabeled in the tables.
b. Poor test items. The objective is judged to be minimal but the item(s) have test construction flaws which probably contributed to a lower level of attainment. Also, an item may not have been a valid measure of an objective.
c. Inappropriate. The writers feel that the objective, as currently written, is not a minimal objective.

Three objectives on the Grade 4 test and 4 objectives on the Crade 7 test have been "flagged" by the writers. For instance, the test items for Objective 16 on the Grade 4 test were judged to be more difficult than required by the objective. Details are given in Section III along with the interpretation of the results.

Thirteen of the 30 objectives tested were mastered by nearly all beginning fourth graders. Eighty-six percent of the "end of grade 3" objectives tested were mastered by at least 70% of the students. Concepts and skills at the end of Grade 6 apparently were a little more difficult. Ten of the 40 objectives were mastered by nearly all beginning seventh graders. Fifty-two percent of the "end of grade 6" objectives were mastered by at least 70% of the students.

As with the 1973 test, the MCTM has considered 85% attainment as an acceptable level representative of "nearly all students".

Assuming that the test items are appropriate, low levels of attainment could be due to the fact that students do not have the requisite skills at the time of the testing. This in turn could be due to either the fact that the objective is not appropriate, i.e., not minimal, or that the students have not received appropriate instruction. Teachers at the past two assessment conferences have supported the view that, with a few minor exceptions, the tested objectives are minimal and that levels of attainment will improve when proper teaching practices are applied. See Section III for suggested changes to objectives which are felt to be inappropriate in their present form.

III

RESULTS AND SUGGESTIONS

The results of the 1974 Grade 4 and Grade 7 tests are reported in the following pages. Remarks directed at objectives and/or test items which were not changed from 1973 are brief. The writers have maximized the space given to teaching suggestions and further diagnostic questions. We encourage teachers to conduct further diagnosis, by objective, with students who have not attained the objective. Many times wrong answers can be traced to simple misconcepts. It may only take a few key questions to uncover the difficulty and one or more related corrective measures to help a student get back on course. While it is not always that easy, good diagnostic instruction can help cut down on the need for later remedial mathematics classes.

The objectives are grouped in sections according to the content outline used in MPOMEM. The layout for reporting the statewide result, the objective description, and a facsimile of a test item are presented on the next page. Interpretive remarks, diagnostic questions and/or teaching suggestions have been included within this section for each objective.

EXPLANATION OF MATERIAL GIVEN IN THE OBJECTIVE AND RESULTS COLUMN

ⓐ ⓑ ⓒ ⓓ

ⓐ → 1. AR-I-A-6 (Grade 4) 92%

ⓔ → **Indicate Objects That Are Same Size**

ⓕ → **Items 56-60**

ⓖ → **Given a set of objects, the learner will recognize objects that are the same size.**

a. The objective identity number assigned for this test.

b. The objective's full code number as it appears in MPOMEM.

c. The objective is to be mastered by the end of Grade 3 and was tested at the beginning of Grade 4.

d. Ninety-two percent of all students tested got either 4 or 5 out of 5 test items correct for this objective. See APPENDIX A to see the percents for zero correct, 1 correct, 2 correct, 3 correct, 4 correct, and 5 correct.

e. The short title for the objective as given on the MEAP data reports.

f. The item numbering for the five test items.

g. The description of the objective being tested.*

---

*Several of the minimal performance objectives were rewritten by the MEAP staff into a form more appropriate for paper and pencil testing in a group administered multiple choice format. This revised version is given in Section III. The reader should review the wording of the original objectives in Minimal Performance Objectives For Mathematics Education In Michigan (MPOMEM).

The writers recognize the difficulties inherent in group administered multiple choice tests. We hope that teachers will continue to use concrete objects for both instruction and individual assessment. We further urge that MEAP not alter any objectives from the concrete to the picture-identification format when such an alteration changes the nature of the skill or concept involved.

PRE-NUMBER/NUMERATION

Classification

1. AR-I-A-6 (Grade 4) 92%
Indicate Objects That Are Same Size
Items 56-60

Given a set of objects, the learner will recognize objects that are the same size.

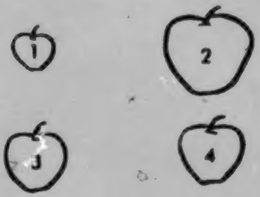
Which of these apples are the same size?

F 3 and 2

G 3 and 1

H 3 and 4

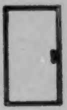
J 1 and 2

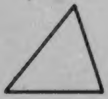


2. AR-I-A-8 (Grade 4) 96%
Indicate Similar Geometric Shapes
Items 51-55

Given an object shaped like a circle, triangle, square, or rectangle, the learner will choose the shape the object represents.

Which figure below has about the same shape as this door?





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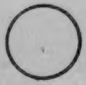

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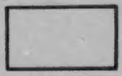

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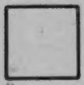


J

1-1 Student performance was quite good on this objective. A minor reading difficulty with these items could be reduced if the nouns were deleted and the words same and size were underlined.

1-2 Students having difficulty with this objective should begin working on classifying real objects on the basis of size. Check each student's understanding of the vocabulary of size comparison. Ask: Which is largest? Which is smallest? Show me how you would find one that is the same size as this one?

1-3 A suggested activity involves real objects (like drinking straws of various lengths) and pictures of these objects. The student picks up one object (one of the straws) and matches it with the corresponding pictures.

He or she is to find the picture of the object which is the same size as the real object.

2-1 The performance on this objective was very good. The Grade 4 students did not perform at a higher level on any other mathematics objective. Ninety-two percent got all five items correct.

2-2 Check to see if the student is confusing same shape with same size.

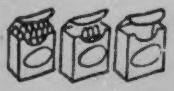
2-3 A student unable to attain this objective may be having a reading or vocabulary problem. Cut some shapes (squares, rectangles, circles, triangles) from heavy paper. Cut some shapes from magazine pictures — for example, a house window, automobile tire, Christmas tree, etc. Ask the student to match the shapes with the magazine cutouts.

Ordering Things

3. AR-I-A-16 (Grade 4) 75%
Indicate Objects Arranged Full to Empty
Items 91-95

Given a set of three containers, one full, one empty, and one half-filled, the learner will choose the containers that are arranged from full to empty.

Which picture shows the crayon boxes in order from full to empty?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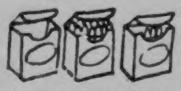
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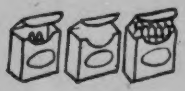
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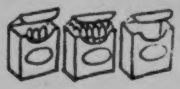
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3-1 The results are less than satisfactory on this minimal objective. A major contributing factor in this low level of performance may be a lack of understanding of the vocabulary: "in order from full to empty". It may also indicate that primary students are not receiving enough experience with ordering real objects. This may, in turn, be a factor in the more general overall weakness involving ordering and comparing numbers.

Pictures do not always convey the intended idea to the child. It might be better to use pictures of glasses, cups, and bottles and to have these containers illustrated in natural settings such as on a table or counter top.

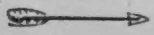
3-2 Have the students draw drinking glasses--full of pop, empty, half full. Have them cut these drawings out and paste them up in order from full to empty. Ask: Which picture will come first? Which picture will come last? Which picture will come between first and last? The half-filled glass will come right after which glass? The half-filled glass will come just before which glass?

3-3 Paste pictures of full, half full, and empty containers on separate cards and have the students arrange the cards in order. Help them connect the action of placing things in order with pictures showing things in order. Place the vocabulary of "in order from full to empty" on the spelling list.

4. AR-I-A-24 (Grade 4) 92%
Indicate Longest and Shortest Objects
Items 116-120

Given a collection of five objects of varying lengths, the learner will identify the longest or the shortest, as requested.

Which arrow is the longest?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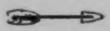
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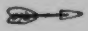
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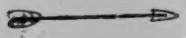
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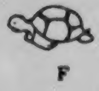
4-1 The level of performance on this objective is quite good. Eighty-six percent got all five items correct. The error most often made was choosing the opposite, e.g., shortest when longest was correct and vice versa.

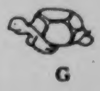
4-2 Instruction on this objective might initially emphasize the term longest first. Use real objects. Interpret the meaning of the suffix "est". Have the student tell, in his or her own words, that he or she cannot find a longer object in the set. After longest is comprehended, then begin to emphasize shortest. Emphasizing one of the pair of opposites at a time may help alleviate the confusion of terms.

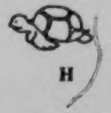
5. AR-I-A-39 (Grade 4) 91%
Indicate First and Last
Items 111-115

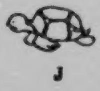
Given five small toys in a line, the learner will identify the first toy and the last one.

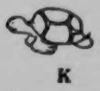
Which turtle is last?











5-1 The results are satisfactory. Those students who made an error usually picked the opposite, e.g., last for first and vice versa. Young children often are able to perceive first from last but they cannot connect the written work with their perception.

5-2 Review the rules for judging which is first: when objects or pictures in a line have a front (car) or a face (cat) they are ordered by the way they are facing. When objects do not have an obvious front or face (box), they are ordered from left to right.

5-3 Children need more concrete experiences with the vocabulary of arranging in order and comparing. They especially need help in making the transition from concrete experience to reading and processing the vocabulary.

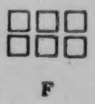
Pre-Number Meaning

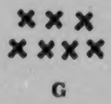
6. AR-I-B-7 (Grade 4) 83%
Choose Equivalent Sets
Items 101-105

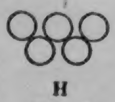
Given a set with less than ten objects, the learner will identify an equivalent set.

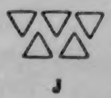
Which group below has the same number of members as this group?

× × ×
× × ×



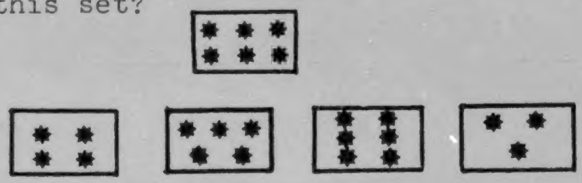






6-1 The results on this minimal objective are barely satisfactory. There is no doubt that nearly all students would do well on this objective if it were measured with sets of objects and the questions were administered orally. The items could be improved slightly along the following lines: underline same number, draw boxes around the sets, use the word set instead of the word group, use pictures of like objects for the choices.

6-2 Check the students' understanding of "same number of members" by using one-to-one correspondence. Encourage them to draw lines connecting the members of the two sets being compared. Ask: Which set has the same number of members as this set?



7. AR-I-B-32 (Grade 4) 73%
Choose Sets Having Fewer Numbers
Items 96-100

Given a set of two to eight objects, the learner will identify a set having fewer members than the original set.

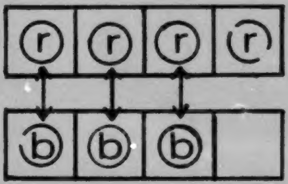
Which group has fewer members than this group? XXX X
XXXX

XXXX XXXXX	XXX XXXX	XXXX XXXX	XXXXX XXXXX
F	G	H	J

7-1 The level of attainment for this minimal objective is less than satisfactory. Fifteen percent missed all five items. The most common incorrect choice (from 9% to 11%) involved selecting the set which had the most members. These students may have a concept of fewer which involves comparing many to few. Also, the order of the comparison may be reversed in their minds. That is, the set in the question may be viewed as having fewer members than the set selected from the available choices. The items could be revised to use the word "less" in place of "fewer". The sets of objects should be "boxed" to make it easier to compare the choices with the set in the question.

7-2 Test your students using the word "less" in place of the word fewer. Interview students who do not respond correctly. Use sets of objects. Ask: Which set has more members than this set? Which set has the same number of members as this set? Be definite about the set to which you are referring. Ask: Which set has less than this set?

Mapping in a one-to-one correspondence shows which set has more, and which fewer or less.

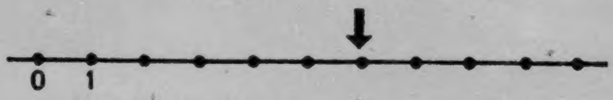


Order/1-digit Numbers

8. AR-I-B-40 (Grade 4) 78%
Indicate Appropriate Numeral For Point On A Line
Items 131-135

Given a line marked with congruent segments and a set of number cards (0-10), the learner will choose the appropriate number card for the point on the line.

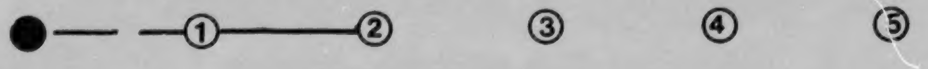
Which number goes with the point marked by the arrow?



F 4 G 5 H 6 J 7

8-1 The results for this minimal objective are less than satisfactory. The incorrect choices show at least two problem areas. First, where available as a choice, from 9% to 11% of the students appear to have been selecting the choice nearest the arrow - ignoring the number line altogether. Of a more critical nature, where offered as a choice, from 8% to 14% appear to have been counting the points on the number line.

8-2 Students can be taught to operate successfully with the number line if the initial concept of a unit interval is developed carefully. Have the students build and label their own number line using materials which will emphasize the unit interval and not the dots. Tinker Toy sticks or drinking straws could be used. Lay a straw (or draw a path) from the origin dot to Dot 1, a second straw from Dot 1 to Dot 2, another to Dot 3, etc. Later, children lay a unit straw along a line, mark off points and number them as they proceed.



9. AR-I-B-43 (Grade 4) 90%
Choose Greatest and Least Number
Items 26-30

Given any three numbers, 0-10, the learner will identify which number is the greatest and which is the least, on request.

Which number is the least?

A 4

B 3

C 6

10. AR-I-B-44 (Grade 4) 92%
Choose Number Between Two Numbers
Items 76-80

Given two consecutive even or odd numbers, 0-9, the learner will name the number that comes between the two given numbers.

Which number comes between 3 and 5?

F 2

G 4

H 6

J 8

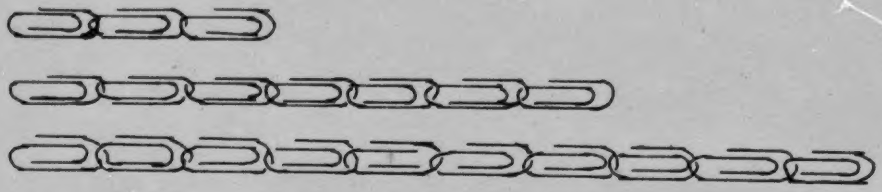
9-1 The level of attainment is satisfactory. With a little attention and maintenance, it could be higher. The most common error involved selecting the greatest number when the least was requested (5% to 7%).

9-2 Check the student's understanding of the vocabulary. Help the students to connect the concept of greatest with most and the concept of smallest with least.

9-3 Evidence here and on some of the other related objectives indicates that there may be a general need to provide more experience with the vocabulary of ordering and comparing things and numbers. See Objectives 3-11, 13-17. A student failing any one or more of these objectives should be interviewed regarding all of these objectives.

10-1 This result is quite good. Eighty-eight percent of the students answered all five items correctly. The most common error, but at a very low level (3%) was to respond with the number that comes just before the lesser of the two given numbers.

10-2 Work individually with students on vocabulary and the concept of ordering 3 things as to length. Ask: Which straw is longer than this one? Which is *longest*? Which straw comes *just before* the *longest*? Which straw is *shortest*? Which straw comes *right after* the *shortest*? Which straw comes *between* the *shortest* and the *longest*? This can be done with paperclips that are linked together to form chains. Numbers can easily be associated with the lengths through counting the number of clips in each chain.



11. AR-I-B-45 (Grade 4) 82%
Choose Number Before or After a Number
Items 81-85

Given a number from 1 to 8, the learner will identify the number that comes *before* or *after* the given number.

Which number comes right after 8?

A 5

B 6

C 7

D 9

11-1 This result is not satisfactory and could be much better. Only 69% got all five of these seemingly simple items correct. The range of correct responses on the three "just before" items was 90-92. For the two "right after" items the results were 81% and 85%. On these two "right after" items, 16% and 13%, respectively, selected the "just before" response. This pattern may have been conditioned by the first two items which were the "just before" types.

11-2 See Remark 10-2. Regular attention to this concept and the vocabulary in the primary grades will yield improvement with this objective.

13. AR-I-B-65 (Grade 4) 71%
Indicate Number Before or After Number Within A Decade
Items 71-75

Given a set of sequentially ordered whole numbers within a decade less than 100, such as 31, 32,...40, the learner will identify the number that comes immediately before or after a given number, as requested.

Which number comes just before 22 in the series (20,21,22,23,24)?

A 20
B 21
C 23
D 24

13-1 This result is less than satisfactory, but somewhat understandable when the results on Objective 11 are taken into account. The performance on "just before" items is at the same level (from 79-80%) as on the "right after" items (81% and 83%). The most common error (11% to 16%) involved selecting the opposite response.

13-2 Print five 2-digit numbers in a box on the chalkboard. For example $\boxed{52, 53, 54, 55, 56}$. Ask questions like: Which number comes last? Which number comes between 53 and 55? Which number comes just before 55? Which number comes right after 53? Fifty-three comes just before which number?

Give the student 4 of 5 cards with a number printed on each card. Hold the fifth card and ask: Which number comes just before (or right after) this number? Check to see if the student understands order relative to value versus the physical position of the printed numerals.

14. AR-I-B-67 (Grade 4) 92%
Indicate Which of 2 Numbers is Greater or Less
Items 31-35

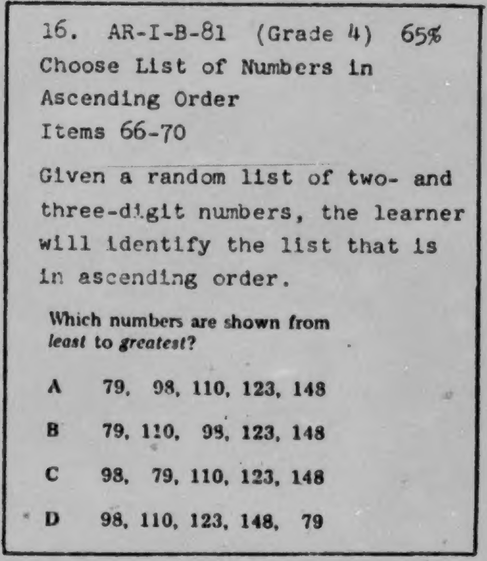
Given 2 two-digit numbers, the learner will tell which number is greater and which number is less.

Which number is greater?

A 29
B 31

14-1 The level of attainment is satisfactory. Nearly all students should do well with greater and less by the end of Grade 3. Seventy-nine percent responded correctly to all five items. Guessing could be more of a factor here since there were only two choices for each item. There was no significant difference between the correct responses for "greater" and correct responses for "less".

14-2 A student who responds that 75 is less than 69 may be comparing the ending sound "five" in seventy-five to the beginning sound "six" in sixty-nine. Check your students with a selection of comparisons to see if you can pick up which part of the stimulus they may be attending to.



16. AR-I-B-81 (Grade 4) 65%
Choose List of Numbers in Ascending Order
Items 66-70

Given a random list of two- and three-digit numbers, the learner will identify the list that is in ascending order.

Which numbers are shown from *least* to *greatest*?

A 79, 98, 110, 123, 148

B 79, 110, 98, 123, 148

C 98, 79, 110, 123, 148

D 98, 110, 123, 148, 79

16-1* This was one of the three lowest levels of attainment on the Grade 4 test. It is interesting to note that while 90% (see Objective 9) could correctly identify which of three numbers was least (or greatest) only 65% could identify numbers arranged in order from least to greatest. On Objective 28, 81% could identify the greatest (least) amount of money.

Some of the error (8% to 10%) appears to come from selecting the reverse order - greatest to least. Some of the error may be due to carelessness. Particularly on three items, many of the students selected a choice where only the last two numbers were out of order. For example, on Item 67, 11% selected D. 98, 110, 123, 148, 79. While the objective is minimal, perhaps four sets of from four to five numbers each is too much on a test.

While this is a minimal objective, it should be emphasized that it is a difficult and complex skill. There are a large number of mental operations which must be made in comparing 3-digit numbers in sets of four or five numbers each. The hundreds digits need to be compared, tens digits, etc. Similar to alphabetizing, many students get lost in the midst of their comparisons.

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*This was objective 17 on the 1973 test. See Appendix B for details.

16-2 Test your students with three sets of three to four numbers each. Start with numbers which are easily compared, e.g., 17, 92, 578 and work toward finer discriminations. Check to see each student can: identify 17 as the least number — it should come first; identify 578 as the greatest number — it should come last. Check to see if the student having difficulty can order paper clip chains by length from shortest to longest. Or if he/she can tell which number in a series comes just before or right after a given number. See remark 13-2. Have the student order number cards from least to greatest.

17. AR-I-B-82 (Grade 4) 89%
Indicate Greater Or Less/
Scrambled Positions
Items 36-40

Given 2 three-digit numbers which have the same digits but in different positions, the learner will compare them to determine which is greater and which is less.

Which number is less?

F 776

G 767

17-1 The results for greater/less with 3-digit numbers is comparable to that for 2-digit numbers - see Objective 14. Only 88% selected 767 as the correct answer for item number 40. Otherwise the percent correct per item for objective 17 averaged about 92%. The fact that 776 can be easily confused with 767 combined with the fact that Items 38 and 39 were both "Which number is greater" questions may account for the lower performance on Item 40.

18. AR-I-B-84 (Grade 4) 80%
Indicate Next Number in a
Sequence
Items 41-45

Given a counting sequence of two or four numbers, the learner will write the next number in sequence.

Which number comes next?
3, 6, 9, ____

A 10

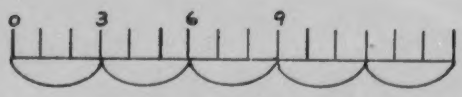
B 11

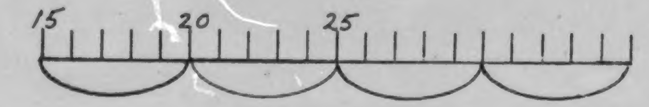
C 12

D 15

18-1 The results are not quite satisfactory considering the low level of difficulty implied by the objective. The writers feel that the "which number comes next" pattern-sequence type of item is not an adequate measure of the objective. The question, "Which number comes next?" is too ambiguous. There were two common errors shown by the choice selection. On the average, by item, about 9% selected the literal next whole number. In Item 43, 9% selected 10. On Items 43 and 45 about 12% selected the response which is the last number in the sequence plus 2. Perhaps they counted by twos and perhaps this was conditioned by Item 41 in which the sequence was 2, 4, 6, ___.

18-2 Skip counting is an important readiness skill for multiplication. It would be helpful to test your primary students to see if they can count by 2's, 3's, 5's, and 10's. Give the sequence 3, 6, 9, ___ and say: Look at these numbers, if you were counting by threes, what number comes next? Give 15, 20, 25, ___ and ask: If you were counting by fives, what number comes next? Extend this activity by writing a counting sequence of four numbers and ask the students to identify the pattern.





Money

15. AR-I-B-70 (Grade 4) 84%
Indicate The Values Of A Set Of Dimes and Pennies
Items 146-150

Given a set of dimes and pennies valued between 11 and 99 cents (one dime, one penny to nine dimes, nine pennies), the learner will state the value.

How much are 5 dimes and 6 pennies worth?

F 55¢
G 56¢
H 65¢
J 66¢

15-1 The level of performance is satisfactory. The attainment would be slightly higher if coins and money examples were more frequently used in the classroom. There were no detectable error patterns. Three items involved pictures of coins (the performances were 87%, 87% and 85%) and two items were like Item 148.

15-2 It is difficult to use dimes and pennies to illustrate and reinforce tens and ones in numeration if the student doesn't understand the money. Check your students understanding of the worth of dimes and pennies. Use real coins or good facsimiles. Obtain a set of cards with a facsimile of various coin combinations on each card. For example: ten cards showing from 1 to 10 pennies; ten cards showing from 1 to 10 dimes, eighty-one cards showing from one dime and one penny to 9 dimes and 9 pennies. These cards can be used in drills, games and as a handy format for various problem applications.

Number Meaning

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19. AR-I-B-65 (Grade 4) 713
Indicate a Number That is A Multiple of 2
Items 61-65

Given the counting numbers 1-10, the learner will indicate those that are multiples of 2.

Which group has *only* even numbers?

A 2, 3, 4
B 2, 6, 8
C 4, 5, 6
D 6, 8, 9

19-1 The level of attainment is less than satisfactory. Seventeen percent of the students answered no more than one item correctly. The objective does not call for the recognition of the term "even". It is reasonable, however, to expect that by the end of Grade 3, nearly all students would be able to identify an even number. Two of the five items involved identifying a set of three even numbers. The level of performance was higher for the three items requiring only the choice of a single even number.

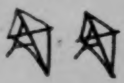
19-2 The implication from the results is that approximately 30% of the beginning Grade 4 students do not know the term <u>even</u> number. The terms even and odd should be recognized and understood by nearly all students by the end of Grade 3. Emphasizing multiples of 2 makes a valuable contribution to subsequent development with multiplication. Have the students group objects in two rows. Even numbered arrays come out even, i.e., two equal rows. Check to see who can count by twos. Work on <u>doubling</u> to make <u>even</u> numbers. When the two addends are equal, then the sum is a double - and an even number.

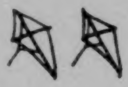
```
   3    □ □ □            3    □ □ □
+  3    □ □ □         +  4    □ □ □ □
   6  Comes out even     7  Hmm! That's odd.
```

20. AR-I-B-86 (Grade 4) 52%
Select Set With Twice As Many Members As Another
Items 121-125

Given a set of objects, the learner will select another set that will have twice as many objects.

Which group below has twice as many kites as this group?





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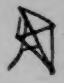

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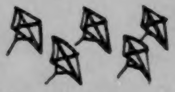

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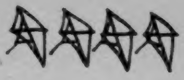
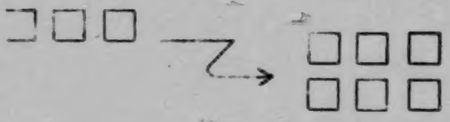


J

20-1 (This was Objective 21 on the 1973 test) The terminology "twice as many" is not familiar to beginning Grade 4 students. An apparently common misconcept connected with the phrase twice as many seems to involve the idea of "lots more". In three items the greatest number of members in the sets offered was selected by from 20% to 22% of the students, i.e., twice as many means lots more. In the two items, the greatest number was twice as many and the level of performance was correspondingly higher on these two items. It is recommended that the objective and test items be changed to "two times as many".

20-2 Test your students using the terminology "two times as many" instead of "twice as many". For those that fail, try the following. Develop the idea of building two rows to show 2 times as many. Say: Build me a set that has two times as many blocks as this ...



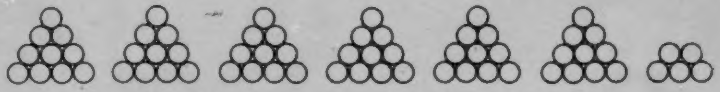
  3 Three one time
+ 3 Three two times
  6 two times 3 is 6

Connect the concept of two times as many with doubling and with multiples of 2.

12. AR-I-B-64 (Grade 4) 86%
Identify A Numeral Less Than 100
Items 121-125

Given a set of tens and ones representing a number less than 100, the learner will identify the numeral.

How many balls are in this group?



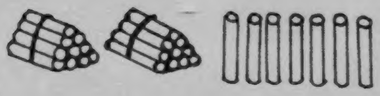
F 45 G 56 H 65 J 74

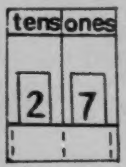
12-1 The level of performance is satisfactory. Some students may have counted all of the balls in each item. Hopefully, most counted the sets of ten and counted the singles left over separately.

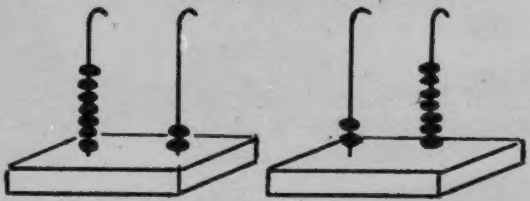
12-2 Test your students with bundles of sticks (popsicle, swizzle, tongue depressors, etc.). Check to see if he or she can accept 7 bundles of 10 as 70 or does he or she need to count them out as seventy singles. Check to see if the student can: count by tens; tell you how many groups of ten in 50, in 80, in 85.

Have the children count: ten, twenty, twenty-one, twenty-two, twenty-three, etc.

Ask: Which means seventy-two, 7 tens 2 ones or 2 tens 7 ones?







1. AR-I-B-87 (Grade 7) 77%
Identify Number 100/1000 Larger
Items 6-10

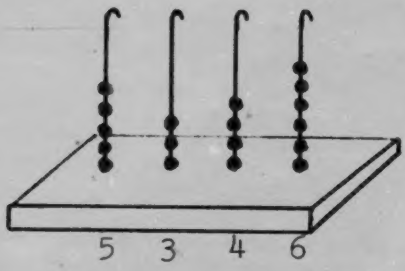
Given any four digit number, the learner will identify the number that is 100 or 1000 more or less than it is, without using formal addition or subtraction.

What number is 100 *more* than 2,746?

- **A** **2,645**
- **B** **2,755**
- **C** **2,846**
- **D** **3,746**

1-1 The results for this minimal objective are less than satisfactory for end of Grade 6. On three of the items involving 100/1000 more, from 11 to 22 percent of the incorrect responses involved the wrong place value. For example, in Item 9, 15% selected 3,746 for 100 more than 2,746. Whether any of the students were attempting to add in the "less" items when they should subtract and vice versa for the "more" items is not clear because the choices did not allow for this. Nor is it clear to what degree students attempted to respond incorrectly using "less" when the request was for "more".

1-2 It is not clear why the students did not perform better on these items. Check to see if your students can explain that 1,000 less than 5,346 means 1 less in the thousands place, i.e., 4,346. Print the digits in place value columns. Use an abacus and connect the terminology 1,000 less with removing one bead on the thousands place wire; adding one bead for 1,000 more. Similarly, for 100 more and 100 less.



Show me what 1,000 less than 5,346 means on this abacus (or counting frame). Do you put a bead on or take a bead off? In which place?

2. AR-I-B-89 (Grade 7) 95%
Identify Arabic Numeral
Items 1-5

Given a number orally, the learner will identify the arabic numeral.

The test questions were read to the students. For item Number 5 the teacher read: Choose the numeral that represents ONE THOUSAND SIXTY-NINE. The students had to find and mark their answer.

A 169
B 690
C 1,069
D 10,069

2-1 These results are quite good. Teaching to read and write numbers is important and the results here are encouraging.

2-2 Five percent of the students missed 2 or more items. It is important to further test your students with some common error producing numbers such as one thousand fifty (1,00050) or eight hundred nine (8009). Record these and other numbers on tape cassette and let the students practice transcribing them.

ADDITION AND SUBTRACTION WITH WHOLE NUMBERS

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21. AR-II-A-10 (Grade 4) 96%
Add Two-Digit and One-Digit Number/No Carrying
Items 11-15

Given addition exercises involving a two-digit number plus a one-digit number requiring no regrouping (carrying), the learner will find the sums with or without the use of aids.

$$\begin{array}{r} 32 \\ +\ 4 \\ \hline \end{array}$$

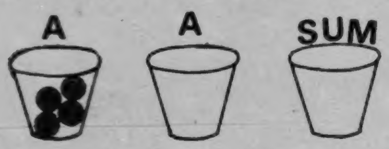
A 36
B 38
C 40
D 46

21-1 Congratulations to teachers and children. 96% was the highest percentage correct for any tested objective. Regular maintenance should keep performance high.

21-2 For those children who did not succeed, try to determine if the difficulty was a lack of fact knowledge especially with zero, or a lack of "sign" recognition. Ask: How much is 0? When nothing is added will there be more?

21-3 For teaching addition with zero, pour the amount in each ADDEND glass into the SUM glass. Finish the number sentence.

4 + 0 = ☐



Recognizing signs: In each sentence write in the true sign:

6 ___ 4 = 10   7 ___ 7 = 0   7 ___ 7 = 14   0 ___ 4 = 4   6 ___ 3 = 9

3. AR-II-A-22 (Grade 7) 94%
Add 3- and 1-, 2-, and 3-Digit Numbers
Items 21-25

Given addition exercises involving a three-digit number plus a one-, two-, or three-digit addend, with or without regrouping (carrying), the learner will identify the sums, using any techniques.

$$\begin{array}{r} 434 \\ +\ 268 \\ \hline \end{array}$$

A 692
B 702
C 711
D 712

3-1 The high percentage of attainment indicates that skill with the addition algorithm is maintained well through the elementary grades. Instructional efforts with addition of whole numbers in Grades 5-9 could well be on a more individual basis.

In several items, selection of an incorrect answer was probably due to failure to regroup in the TENS and/or HUNDREDS place(s).

3-2 Ask: "In what place do you add first? If the sum is less than 10 what do you do? If the sum is greater than 10 what is the procedure? Will you need to rename a ten when both addends are less than 5? When both addends are 5 or greater than 5? Why? Suppose only one addend is greater than 5 -- is it possible to have a sum of 10? Name some combinations where this is true (3 + 7, 8 + 2, etc.)

3-3 Practice having one child suggest the first addend and another child supply a second addend sufficient to make regrouping necessary. Everyone works the problem to determine if specifications have been met. Specify some problems that will not require regrouping, and some that require regrouping in both places.

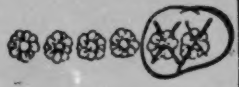
First child: 2 8
Second child: + 1 3

First child: 3 7
Second child: + 7 9

22. AR-II-B-9 (Grade 4) 67%
Number Sentences/Subtraction
Items 136-140

Given a set of objects or pictures showing a subtraction relationship with combinations to 18, the learner will identify an appropriate number sentence.

Which number sentence below tells about this picture?



F 4 − 2 = 2

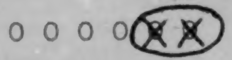
G 6 − 2 = 4

H 2 + 6 = 8

J 8 − 6 = 2

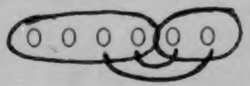
22-1 It is important to teach children to draw and interpret pictures. The great number of incorrect responses to each of the five items on both the 1973 and 1974 tests indicate that the children could not relate the picture to the correct equation. 18% missed all five items on the 1974 test.

It appears that most children who selected incorrect answers interpreted the picture not as one set of objects with a subset removed (separation subtraction),



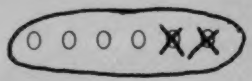
6 - 2 = 4,

but as two sets of objects to be compared (comparison subtraction),



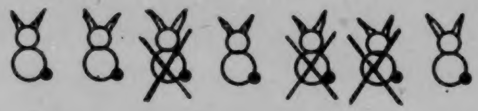
4 - 2 = 2.

22-2 Test the child's recognition of the whole set by asking him or her to draw a ring to show the whole. Further test this objective using diagrams such as:



Produce pictures similar to those in the test, and ask the children: What does crossing out a member of the set mean to you? How many were in the set of flowers before some were crossed out? How many were crossed out? How many were left?

22-3 Teach the children to identify the whole and then the known or given part. A child draws a set of simple objects on the chalkboard, and invites a classmate to tell how many in all and to select and X out 3 of the objects and write the equation:



7 - 3 = 4

The X-out members may or may not be circled. The important concept is the idea that "crossing out" means discarding or subtracting objects from the set.

23. AR-II-B-11 (Grade 4) 84%
Number Sentences/Addition or Subtraction-Identify Operation - Items 1-5

Given a subtraction word problem read by the teacher involving combinations to 18, the learner will: 1) identify the operation, 2) identify an appropriate number sentence, and 3) identify the answer.

Toby is seventeen years old. His sister is eleven years old. Which letter answer tells how much older Toby is than his sister?

A 17 + 6 = 23
B 11 − 5 = 6
C 6 + 7 = 13
D 17 − 11 = 6

23-1 The word problems were read by the teacher and contained simple words. Clearly, the difficulty was not due to reading or word meaning, but rather to a visualizing of the ideas contained in each problem.

23-2 To determine if children have developed the ability to understand, think through, and visualize a word problem, the teacher reads a word problem to the class only once, and asks detailed questions:

Sue had 11 pennies. Sam had 9 pennies. How many more pennies did Sue have than Sam?

What was the girl's name? The boy's name? What did they have? How many did Sue have? Sam? Did they have the same number of pennies? Who had more? What did the question ask? What is the answer? Draw a picture of Sue's pennies. Draw a picture of Sam's. Which would you rather have? Why?

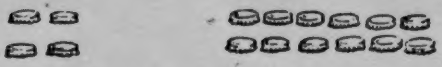
23-3 To help develop the child's visualizing ability, a different problem is read to the children every day for many weeks, and detailed questions are asked. Children are asked to select the correct number sentence for each problem from a set written on the chalkboard, or write one that explains the problem in numbers. Try tape recording a set of story problems. The detailed questions could also be taped or printed on 5 by 7 cards.

24. AR-II-B-13 (Grade 4) 73%
Numerical Set Comparisons
Items 126-130

Given two sets of objects, one with more than the other, the learner will identify how many more members it has.

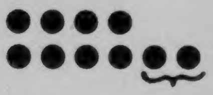
How many *more* bottle caps are in the larger group than in the smaller group?

F 4
G 7
H 8
J 12



24-1 The phrase, "than in the smaller group" was added to each item on the 1974 test. In both the 1973 and 1974 tests, the most common error was to identify the large group that was readily seen to contain more members, count the members of this set, and name this number as the answer, instead of completing the comparison idea of telling how many more the larger set contained than the smaller one.

24-2 Comparisons may first be shown by lining up the objects in a spatial-related arrangement, or by a one-to-one drawing of the two sets, and by writing the comparison subtraction sentence. Later, the concrete objects or the pictured sets are not lined up but grouped in an informal manner where the children must count and perform subtraction to find out "how many more". Breaking the problem down into simpler questions helps the child focus on the key parts. Ask: Which set is larger? How many in the larger set? How many in the smaller? How many more in the larger set than in the smaller?



$6 - 4 = 2$

24-3 Realizing that children have an innate sense that it is fair to share equally, a situation may be created where two children receive non-equivalent amounts of candy. Ask: Did John and Dan receive equivalent amounts of candy? Who has the larger amount of candy? How many more pieces would Dan need to have as many as John? When you are asked to tell how many more one child has than another how can you find out?

25. AR-II-B-15 (Grade 4) 92%
Subtract One-Digit From Two-Digit
Number No Borrowing
Items 16-20

Given a two-digit number, the learner will subtract one-digit numbers with no regrouping (borrowing) with or without the use of aids.

$$\begin{array}{r} 27 \\ -\ 4 \\ \hline \end{array}$$

F 20
G 21
H 23
J 25

25-1 The results are very acceptable. The percent correct per item ranged from 92% to 96%.

25-2 To ascertain how the children who did not achieve the objective may be helped, first test for facts mastery. Also, have individuals say aloud what they are thinking as they solve similar problems.

25-3 Practice only a few difficult facts at a time, mixed in with about five facts that are well known. Make a game of learning these. Easy facts score 1 point, while difficult ones score 5 points.

26. AR-II-B-16 (Grade 4) 86%
Subtract Two-Digit From Two-Digit
Number/No Borrowing
Items 21-25

Given a two-digit number, the learner will subtract a two-digit number with no regrouping (borrowing).

$$\begin{array}{r} 98 \\ -\ 73 \\ \hline \end{array}$$

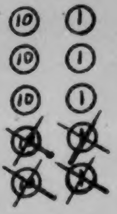
A 25
B 35
C 41
D 52

26-1 The variation of errors makes it difficult to determine the major reason for the errors. Apparently, transposition was a cause of some wrong answers, for 7% of the children selected 52 for an answer that should have been 25.

26-2 Ask: In what place do you subtract first? Where is the answer written? In what place do you subtract next? Where is this answer written? How are the two subtractions alike? Why is it important that the answers be written in the correct places?

26-3 To illustrate the similarity of using the same facts knowledge in both the ONES and TENS places (or in any decimal place), draw discs on the chalkboard, and do the necessary crossing out as the subtraction is performed.

$$\begin{array}{r} 5\ 5 \\ -\ 2\ 2 \\ \hline \end{array}$$



Paper discs (different colors for TENS and ONES) are taped to the chalkboard and removed as the subtraction is performed in each place.

The expanded form is used to review subtraction in the places, first - ONES, second - TENS:

$$\begin{array}{rcl} 7\ 5 & \dashrightarrow & 7 \text{ tens} \quad 5 \text{ ones} \\ \underline{-\ 4\ 3} & \dashrightarrow & \underline{4 \text{ tens} \quad 3 \text{ ones}} \end{array}$$

Place names are said by the child as he works the problem:

> "5 ones minus 3 ones equals 2 ones. 7 tens minus 4 tens equals 3 tens."

4. AR-II-A-26 (Grade 7) 87%
Add Two Or Three Numbers
Items 26-30

Given addition problems involving two or three addends with three, four, five, or six digits, with or without regrouping, the learner will find the sums, using any techniques.

$$\begin{array}{r} 2{,}371 \\ 50{,}452 \\ +\ \ 938 \\ \hline \end{array}$$

A 52,651
B 52,761
C 53,651
D 53,761

4-1 Most errors were due to failure to regroup in the TENS place or in the THOUSANDS place — evidently the same regrouping difficulty experienced in the Fourth Grade has been carried through to the Seventh Grade. These results are satisfactory, but more improvement could be expected after a period of drill and practice. Some of the regrouping errors may have been caused by carelessness. Fifteen percent selected 52,761 on Number 29.

4-2 Similar to diagnosing in Objective 3. Also, ask children to think aloud as they work similar problems. These thoughts may be taped and replayed, giving children a means of diagnosing their own work.

4-3 Since forgetting to regroup is a common error, an adding pattern should be established to help children guard against forgetting. This may include always adding in the same direction — from the top down the column — and to "carry" any tens that must be regrouped to the top of the next higher-place column to be immediately added to the numbers in this place. This pattern is followed throughout the problem.

5. AR-II-B-20 (Grade 7) 82%
Subtract 2- Or 3-Digit Number From
3-Digit Number
Items 11-15

Given a three-digit number, the learner will subtract a two or three-digit number, with or without the use of aids.

$$\begin{array}{r} 800 \\ -\ 277 \\ \hline \end{array}$$

A 522
B 523
C 533
D 677

5-1 Although statewide the achievement was nearly acceptable, 3% of the children missed all five of the items, and 26% failed to subtract correctly from 800. The range of percent correct for the other four items was 84% to 90%.

This is a minimal objective, but results show that help is needed when it is necessary to regroup twice and when zeros are involved.

5-2 To see if children understand when regrouping is necessary, give fact problems, like those shown below, and ask children to work only those that can be answered (using a whole number).

$$\begin{array}{r} 9 \\ -\ 8 \\ \hline \end{array} \qquad \begin{array}{r} 0 \\ -\ 7 \\ \hline \end{array} \qquad \begin{array}{r} 8 \\ -\ 5 \\ \hline \end{array} \qquad \begin{array}{r} 6 \\ -\ 6 \\ \hline \end{array} \qquad \begin{array}{r} 8 \\ -\ 9 \\ \hline \end{array}$$

On two items, 13 and 15 (shown on page 33), a high percent of the students (11% and 8% respectively) selected an incorrect choice which may indicate that they were performing their regrouping in a mindless or mechanical fashion. Check individual children to see if they are making this type of error.

13. $\begin{array}{r} ^{7}\cancel{8}\,^{1}0\,^{1}0 \\ -\,2\;7\;7 \\ \hline 5\;3\;3 \end{array}$ 11%

15. $\begin{array}{r} ^{5}\cancel{6}\,^{1}2\,^{1}5 \\ -\,5\;8 \\ \hline 5\;7\;7 \end{array}$ 8%

They write a one just above each digit in the ones and tens place, then they cross out the digit in the hundreds place and write the number which is one less just above the crossed out digit. Then they subtract in each place.

Using plastic glasses taped to the chalkboard, and sticks bundled in sets of 10's and 1's, ask the child to illustrate the problem by performing the necessary action as he does the written work. (See illustration in 1973 Monograph, p. 19).

Give a problem like,

$\begin{array}{r} 6\;2\;5 \\ -\;\;5\;8 \\ \hline \end{array}$ Ask: Can we subtract in the ONES place? What must we provide to make subtraction possible in the ONES place? Can we subtract in the TENS place? What is necessary to make subtraction possible in the TENS place? How is the work similar for both places?

Include problems involving zeros, such as:

$\begin{array}{r} 4\;0 \\ -\,2\;7 \\ \hline \end{array}$ $\begin{array}{r} 1\;0\;0 \\ -\;\;2\;7 \\ \hline \end{array}$ $\begin{array}{r} 4\;0\;3 \\ -\;\;8\;1 \\ \hline \end{array}$

5-3 For help with Number 13, try the quick method of renaming - place the hand behind the TENS place (covering the ONES place), and see 80 tens instead of 0 tens. This allows 1 ten to be regrouped, leaving 79 tens.

$\overset{7\;9}{\cancel{8}\;\cancel{0}}\,^{1}0$ --- All 80 tens are easily seen; 9 is never written over the ONES place.

$-\,2\;7\;7$ --- Subtraction may be performed in every place without further regrouping.

Note: See the diagnostic test for subtraction in Section V.

MULTIPLICATION AND DIVISION WITH WHOLE NUMBERS

6. AR-II-C-6 (Grade 7) 87%
Represent Repeated Addition As Multiplication
Items 31-35

Given a repeated addition sentence, the learner will represent it as a multiplication sentence with its product.

9 + 9 + 9 + 9 = 36 means the same as which one of the following?

F 9 + 3 = 36

G 36 X 1 = 36

H 31 + 4 = 36

J 4 X 9 = 36

6-1 The level of attainment for this objective, while satisfactory, could be a little higher. Some of the errors were no doubt due to carelessness. Approximately 6% to 8% of the students either selected the wrong operation e.g. 6 + 7 = 42 instead of 6 × 7 = 42 or they selected an unrelated but true sentence, e.g. 36 × 1 = 36 instead of 4 × 9 = 36. Perhaps some of the "end of sixth graders" had forgotten about repeated addition and were confused as to the question, "···means the same as which of the following?"

6-2 Multiplication may be introduced using the equivalent set approach. When sets are not equivalent, they may only be totaled by counting or adding. When sets are equivalent, they may be totaled by counting, adding, or MULTIPLYING.

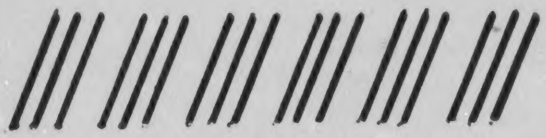
(a) 0 0   0 0 0   0 0 0 0   0   Count
2 +   3   +   4   + 1 =   or Add

(b) 0 0 0   0 0 0   0 0 0   0 0 0   Count,
3   +   3   +   3   +   3 =   or Add
4 × 3 =   or Multiply

Ask: "How many times do you see 3 in (a)? In (b)? Which addition sentence may be expressed as a multiplication sentence? Write the multiplication sentence."

6-3 When multiplication is introduced in the earlier grades have children make equivalent sets of blocks or other objects, and write addition and multiplication sentences to fit the concrete examples.

Conversely, children illustrate multiplication sentences using blocks or other objects. 6 × 3 may be illustrated with sticks as shown:



7. AR-II-C-11 (Grade 7) 94%
Inverse Multiplication
Items 36-40

Given two numbers, the learner will demonstrate that the order in which they are multiplied does not change the product.

Which one of these names the same number as 16 × 12?

F 12 × 16

G 16 × 16

H 12 × 12

J 12 × 13

7-1 While the high level of attainment seems to indicate that the commutative concept is being successfully taught and learned, a great deal of the success may be due to the similarity between the numbers in the question and those of the correct answer — even if a child does not know the concept he will probably choose the correct answer. One of the items contained an answer with a plus sign but with numbers similar to the question, and 5% of the children selected this incorrect answer.

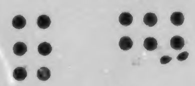
7-2 Determine if the student was merely careless in selecting his/her response.

7-3 Most students understand commutativity by the end of grade 6. For those who do not, here are some teaching suggestions:

(a) Give seesaw problem, e.g., if there are 4 objects each weighing 10 kilograms on one side, and you put 10 objects on the other, what must be the average weight of each?

(b) Draw arcs above a number line to show 8 × 2, and arcs below to show 2 × 8.

(c) Make a rectangular array. Write the multiplication fact. Turn the array and write this fact. Compare.



2 × 3 = 6

3 × 2 = 6

(d) Have the child look at the multiplication chart for the answer to a fact, commute the fact and look for the answer to this new fact. For children learning the facts, the commutative concept is particularly valuable, for it allows him to learn two facts at one time, thus decreasing the learning of 100 facts to 55 facts.

8. AR-II-C-13 (Grade 7) 91%
Multiply 1-Digit Number and Multiple of 10/100
Items 51-55

Given a one-digit number and (10,20...), (100,200...), the learner will identify the product.

8 × 400 = ☐

A 200
B 320
C 1,200
D 3,200

8-1 While 91% achieved success, only 75% were able to answer all 5 items correctly. Errors included ending zero difficulty, fact errors, and adding instead of multiplying.

8-2 Determine whether or not the student understands the shortcut form of multiplying multiples of ten and multiples of 100 by annexing zeros. Have the students verify their answers by repeated addition.

4 × 300 = ☐
Think: 4 × 3 = 12
write 2 zeros after 12,
so 4 × 300 = 1200

Check:
$$\begin{array}{r} 300 \\ 300 \\ 300 \\ +\ 300 \\ \hline 1200 \end{array}$$

Emphasize that multiplying with multiples of ten and 100 is a helpful tool for estimation. For example, to estimate 6 × 283, think: 6 × 300 = 1800.

9. AR-II-C-15 (Grade 7) 87%
Multiply 2-Digit and 1-Digit Numbers
Items 46-50

Given a two-digit number to be multiplied by a one-digit number, the learner will identify the product, with or without aids.

67 × 7 = ☐

F 429

G 462

H 469

J 479

9-1 The objective is minimal and the statewide performance was acceptable.

Only 72% of the children correctly answered all five items. Neglecting to regroup was the major cause of errors. In Item 50, 7% missed 7 × 7, confusing it with 7 × 6, clearly a fact error.

9-2 Show children problems, some requiring regrouping and some not, and ask: In the first problem do you need to regroup? If not, tell the number of tens you will add to your TENS product from the ONES place. (answer 0.) Treat the other problems in a similar manner.

$$\begin{array}{r} 0 \\ 42 \\ \times\ 3 \\ \hline \end{array} \qquad \begin{array}{r} 1 \\ 42 \\ \times\ 8 \\ \hline \end{array} \qquad \begin{array}{r} 6 \\ 48 \\ \times\ 8 \\ \hline \end{array}$$

9-3 Make children aware that when multiplying in the TENS place or any higher place, they will be adding in "some" tens or "0" tens from the ONES place. Fix the idea of always regrouping even if it is zero tens. Use an array to illustrate 2-digit multiplication. See page 72.

Hint for multiplying 7 × 7. Both factors are odd numbers so the answer will be odd. Think 49. When is a product even? Are most products on a multiplication chart even or odd numbers? Why?

10. AR-II-D-5 (Grade 7) 87%
Supply Missing Factor/Multiplication
Items 56-60

Given a sentence with one single digit, a missing factor, and a product (whole numbers), the learner will identify the missing factor.

4 × ☐ = 32

A 6

B 7

C 8

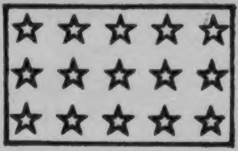
D 9

10-1 While the results are not unsatisfactory, it is hoped that, with some regular maintenance of the facts, the results could be better.

Most confusing facts were 9 × 7, 9 × 6, 8 × 8, and 8 × 7, and related missing factor problems.

10-2 Give a child a missing factor sentence, and ask him to illustrate it using blocks or by drawing a simple picture.

$\square \times 3 = 15$



10-3 Relate four facts, e.g., 4 × 3, 3 × 4, 12 ÷ 3, and 12 ÷ 4. Show how a single simple diagram or array can represent all four at the same time.

11. AR-II-D-7 (Grade 7) 83%
Rewrite Division Fact As Multiplication Fact
Items 86-90

Given a division fact, the learner will identify it rewritten as a multiplication fact.

Which multiplication sentence below below is
is the inverse of 18 ÷ 6 = 3 ?

A $6 \times 18 = 108$

B $18 \times 3 = 48$

C $3 \times 18 = 48$

D $6 \times 3 = 18$

11-1 A common incorrect answer involved replacing the ÷ symbol with × , e.g. 15 × 3 = 75 for 15 ÷ 3 = 5. Knowledge of the meaning of "inverse" is necessary to correctly answer the items on the test. The test items should be rewritten to avoid using the word inverse. It is not used in many textbooks and it is not required by the objective.

11-2 Check to see if the student understands that multiplication and division are inverse operations. Note that the operations are inverses of each other, not the sentences.

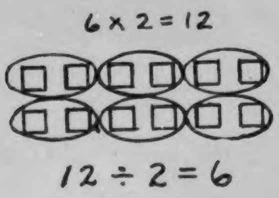
When sets are equivalent we can find out how many altogether by the operation of multiplication.

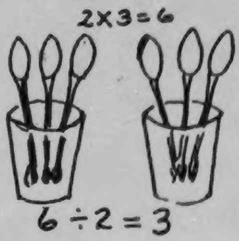
Ask: What inverse operation separates the number of objects altogether back into equivalent sets?

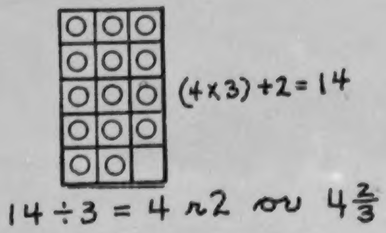
Ask: What is the inverse operation of division?

What is the related multiplication sentence for 12 ÷ 6 = 2? (6 × 2 = 12, or 2 × 6 = 12).

11-3 The inverse operations of multiplication and division may be illustrated by blocks, spoons arranged in glasses, buttons in arrays, or other concrete objects.







Play the "inverse" game whenever there is a free moment during the day. Say: Phil, repeat what I say and then tell the related division sentence, "4 × 8 = 32, and ____________________". (Phil: 4 × 8 = 32 and 32 ÷ 8 = 4.)

12. AR-II-D-9 (Grade 7) 76%
Division/One-Digit Divisor, Dividend Less Than 100
Items 71-75

Given a one-digit divisor (factor) and a dividend (product) of less than 100, the learner will identify the quotient (missing factor) if there is no remainder.

$7\overline{)91}$

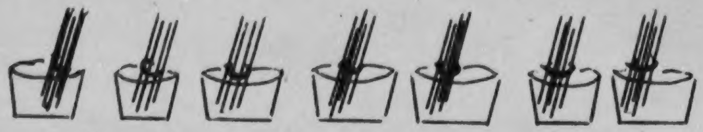
A 11
B 12
C 13
D 14

12-1 This is not a satisfactory level of performance for the end of Grade 6. Poor knowledge of the division facts, of regrouping, and of the division steps, all contributed to the disappointing results.

12-2 Check the student on his/her multiplication facts. See how well the child can estimate the quotient. Use the division problem $7\overline{)9\ 1}$ and ask: Will your answer be less than 10 or greater than 10? How do you know? Will your answer be greater than 20? Explain. If you use 1 ten as the TENS part of your answer how many will you have divided? What remains to be divided? What is the ONES part of your answer?

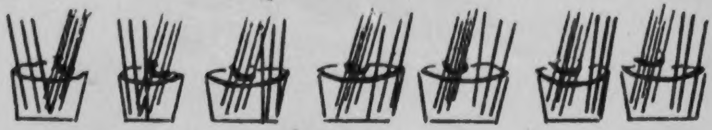
12-3 The equation form and the box form for division may both be practiced at the same time. Also, the division equation and the multiplication equation may be associated to show the relationship between the two operations.

Have the students picture the division question in this manner: The dividend, 91, tells me how many straws I have. The divisor, 7, tells me how many drinking glasses. The division question is, "If I divide these 91 straws among the 7 glasses, how many straws will be in each glass and how many will be left over?"



$$\begin{array}{r} 1 \\ 7\overline{)91} \\ \underline{70} \\ 21 \end{array}$$

I can put 1 group of 10 in each glass — that is 70 straws and leaves 21 yet to be divided.



$$\begin{array}{rl} 13 & \\ 7\overline{)91} & \\ \underline{70} & \\ 21 & \\ \underline{21} & \text{7 threes} \\ 0 & \end{array}$$

Seven times what will equal 21?
How many straws remain to be divided?

13. AR-II-D-15 (Grade 7) 65%
Division/One-Digit Divisor,
Four-Digit Dividend
Items 76-80

Given an exercise with a dividend of four digits or less, and a one-digit divisor, the learner will identify the quotient.

$8\overline{)4{,}112}$

F 2,120
G 480
H 510
J 514

13-1 The poor results were largely due to fact errors and failure to properly distribute the dividend.

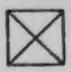
13-2 The teacher may wish to devise a diagnostic test especially suited to his own children. A sample test is included in Section V of this monograph. Plan a course of corrective action for the particular needs of the children based on the diagnostic test.

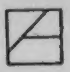
FRACTIONS -- MEANING AND ORDERING

14. AR-III-A-1 (Grade 7) 69%
Identify Congruent Parts
Items 176-180

Given several objects, some divided into congruent parts, some divided into non-congruent parts, the learner will identify congruent parts.

Which figure is divided into congruent parts?


A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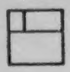
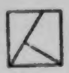

C

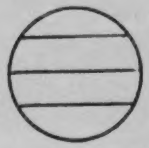

D

14-1 The performance is low on both the 1973 and 1974 tests, considering that equal-size parts is an essential idea for fractions.

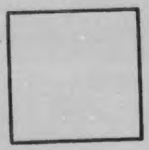
Ten percent of the students did not even get one item correct. Some errors probably occurred because a given circle, square or rectangle was not viewed as a single unit that has been cut into parts. Also, there is probably trouble in recognizing when given parts are equal or not equal in size. While misunderstandings such as these may have been partially responsible for the relative poor performance, use of the word "congruent" is also a likely source of error. Provision for teaching the meaning of this term should be made during the upper elementary grades. However, mastery of the word congruent should not be required for successful work with fractions.

14-2 Use a sheet of paper folded into two congruent parts and ask: Are the pieces congruent? Are the pieces the same size and same shape? Repeat with three equal size parts, four equal size parts, etc.

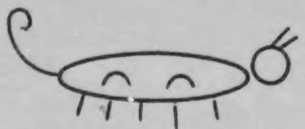
Use a circular coffee filter. Fold it horizontally to show slices of the same width but not of the same area. Repeat the questions.



Use various shapes and ask pupils to draw lines to cut them into two congruent pieces; others into three congruent pieces; etc.

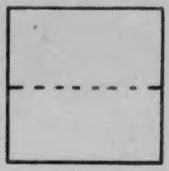


14-3 Draw a figure such as the one shown. Ask: Who can make the best copy of this figure without tracing? Check the results by holding the two figures to the window or to a strong light.

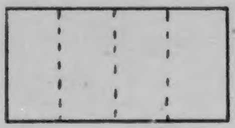


Put "congruent" on your new word list and on your spelling list.

Use paper and fold it to teach equal size pieces, using the word "congruent" as it is done. After folding a sheet of paper into two equal-size parts, say: Can you show me the whole sheet of paper? Can you show me one part? Do the two parts match exactly? If they match exactly, they are congruent.

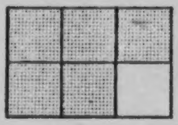


After using sheets of paper to establish the idea firmly and to teach the word "congruent", move to diagrams as "pictures" of the sheets of paper. Draw dotted lines to show where the folds occurred on the paper. Say: Are the two pieces congruent? Repeat with diagrams showing four pieces of equal size.



15. AR-III-A-18 (Grade 7) 70%
Identify Shaded Area Of Figure With Fraction
Items 91-95

Given a diagram divided into congruent parts, with some parts shaded, the learner will identify the shaded area by identifying an appropriate fraction.



A $\frac{1}{5}$
B $\frac{1}{6}$
C $\frac{5}{6}$
D $\frac{6}{5}$

15-1 Since initial fraction concepts tested here are essential for all further learning related to fractions, the results must be considered especially poor. These initial fraction concepts can be learned with relative ease at ages 8 or 9 with proper materials and adequate time allotments.

Several foils for these test items were changed from 1973. In general these incorrect choices drew a higher percent of responses and this probably accounted for the lower percent of attainment (70% versus 76% for 1973).

There are three types of errors shown by the incorrect responses:

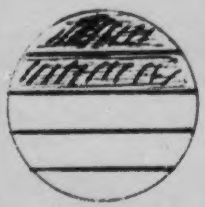
(a) A fraction was chosen to show the part not shaded.. Perhaps the shaded parts were viewed as empty places after these parts have been removed.

(b) A fraction was chosen which shows comparison of the two parts, the unshaded part to the shaded part.

(c) A fraction was chosen to show the smaller part, whether it was shaded or not shaded.

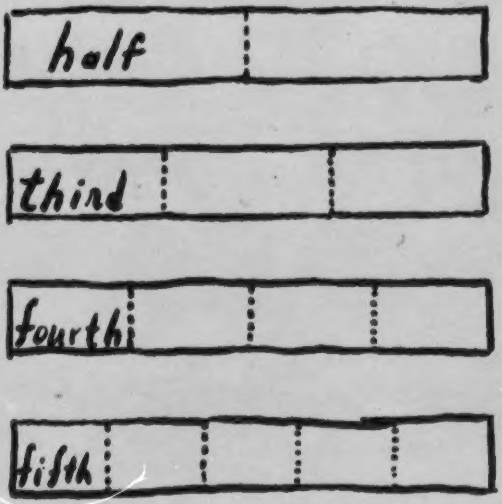
15-2 For further diagnosis, use concrete objects and ask similar questions. Have pupils write the fraction. Check to see if the student is making a part-to-part instead of a part-to-whole comparison. See if some children write $\frac{5}{2}$ when they should write $\frac{2}{5}$, reversing the order of the two numbers in the fraction.

Repeat the question with circular regions cut with parallel slices. Since the parts are not the same size, the fraction $\frac{2}{5}$ does not tell the part shaded.



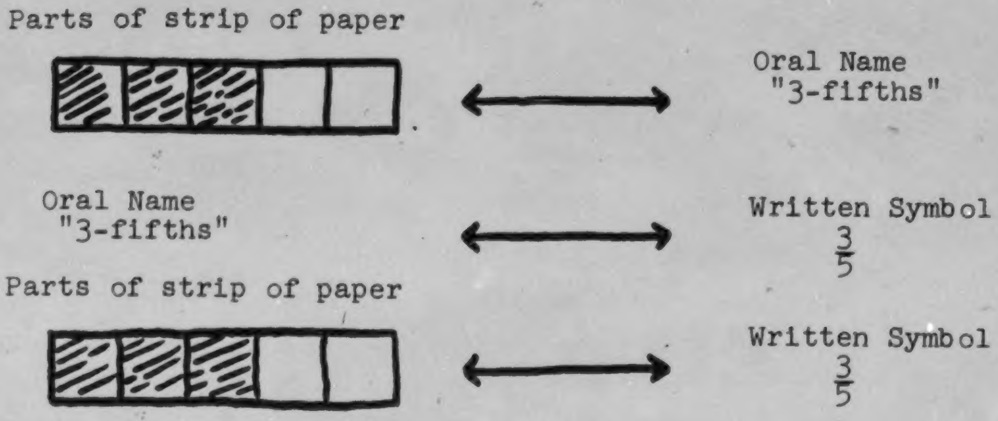
15-3 Have each child make fraction bars from strips of paper 5 cm wide by 20 cm long. Make strips to show halves, thirds, fourths, fifths, sixths, eighths, and tenths. Write a word name for each part on one of the parts. The teacher should make a larger set of models to use for display and to ask questions of the class.

Teach children the word names for fractions first. Stress the need for a) knowing the whole unit; b) using the name for the given part to show the size of one part; c) naming the number of parts and the size of the part displayed, e.g. 3-fifths.



After the word names have been learned well using the fraction bars, introduce the written symbols for fractions. For "3-fifths", you can write $\frac{3}{5}$.

Make sure that instruction includes all these components:



Parts of strip of paper ↔ Oral Name "3-fifths"

Oral Name "3-fifths" ↔ Written Symbol $\frac{3}{5}$

Parts of strip of paper ↔ Written Symbol $\frac{3}{5}$

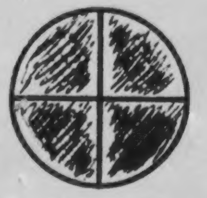
Use a demonstration model of circular units and ask similar questions.

The initial work using strips of paper should be done until pupils master the initial relations between concrete objects, oral name and written symbols. For children ages 8 or 9, this should take about one week.

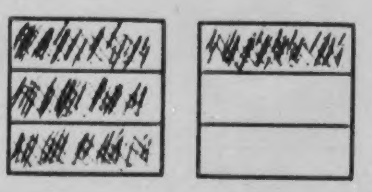
After mastery of the work with concrete materials, make a careful transition to making pictures of the concrete materials using "diagrams". Shading is used to show the pieces of paper to be considered. The pieces left blank are the ones not to be considered. Care must be exercised so that children see in the diagram all the information that we want to convey. Interpreting and drawing diagrams should take 3 or 4 days if the concrete work is done well.

Teach the word "shaded" and add it to the spelling and word lists.

As both concrete objects and diagrams are used, include questions that ask the child to name a whole unit as $\frac{4}{4}$, $\frac{3}{3}$, $\frac{5}{5}$, etc.
Also include examples to show more than one, e.g. $\frac{5}{4}$, $\frac{6}{5}$, etc.



$\frac{4}{4}$



$\frac{4}{3}$

16. AR-III-A-19 (Grade 7) 49%
Order Fractions With Like Denominators
Items 101-105

Given any five fractions with like denominators, in random order, the learner will identify them in order (halves, thirds, fourths, fifths, sixths, eighths, tenths); with or without the use of aids.

Which group of fractions below is in order *of increasing value*?

A $\frac{4}{5}, \frac{6}{5}, \frac{3}{5}, \frac{8}{5}, \frac{2}{5}$

B $\frac{2}{8}, \frac{4}{8}, \frac{3}{8}, \frac{6}{8}, \frac{9}{8}$

C $\frac{1}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}, \frac{7}{6}$

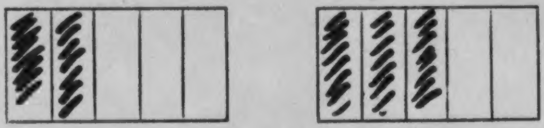
D $\frac{3}{10}, \frac{8}{10}, \frac{6}{10}, \frac{9}{10}, \frac{5}{10}$

16-1 In the 1973 test, the wording was "least to greatest". Changing the wording in the 1974 test to "of increasing value" seemed not to have had a positive effect on performance (56% met criterion in 1973). However, a correct choice for one item was altered from $\frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}$ to $\frac{2}{6}, \frac{4}{6}, \frac{5}{6}, \frac{6}{6}, \frac{8}{6}$. Only 54% got this item correct in 1974 versus 66% in 1973.

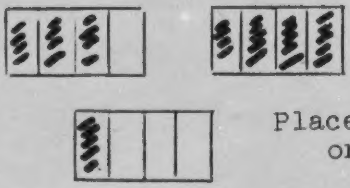
The major incorrect choice was for fractions in decreasing rather than increasing order. This suggests that the language used both years is not well learned by pupils.

Another source of error occurred with two fractions near the middle of a set in reverse order. With so many different comparisons to make, this oversight is understandable.

16-2 Compare regions first, using only two to start with. Say: The cards show two parts of wholes. Put them on the chalkboard rail in order from least to greatest. Now arrange them in decreasing order. Associate decreasing with down.


Place in order

Make other regions using 3 by 5 cards to show parts and do similar comparisons. After making sure all pupils can compare two regions and can use the correct language to describe it, use three cards with regions shown. After mastery with three, increase the number to four and then to five.


Place in order

Repeat the procedures using fractions written on cards, beginning with two fractions, then three fractions, four fractions and then five fractions. When there is trouble with fractions, revert back to diagrams.

$\boxed{\frac{5}{6}}$ $\boxed{\frac{2}{6}}$ $\boxed{\frac{6}{6}}$

$\boxed{\frac{7}{6}}$

Mastery of the content on objectives 14 and 15 is essential for pupils to understand and do well on objective 16.

Put "increase", "decrease", "in order from least to greatest", and "in order from greatest to least" on word and spelling lists.

ADDITION AND SUBTRACTION WITH FRACTIONS

17. AR-III-B-4 (Grade 7) 66%
Identify Sum Of Two Fractions With Like Denominators
Items 116-120

Given two common fractions with like denominators and a sum greater than 1, the learner will identify the sum, with or without the use of fractional cut-out parts.

$\frac{3}{5} + \frac{4}{5} = \square$

F $\frac{7}{10}$
G $\frac{7}{5}$
H $\frac{9}{5}$
J $\frac{4}{3}$

17-1 Addition of like fractions is usually introduced at grade 4 and sometimes in grade 3 and is maintained in subsequent grades. With only 66% meeting criterion by grade 7, and with 15% getting no items correct, overall performance must be considered quite unsatisfactory.

The most common error on the test was adding the numerators to get the numerator of the answer and adding the denominators to get the denominator of the answer.

The results on this objective are hardly surprising, in view of the performance on objective 15. With a strong background of initial knowledge suggested by objective 15, addition of like fractions is very easy. Without that knowledge, addition of fractions becomes the application of a rule with no understanding.

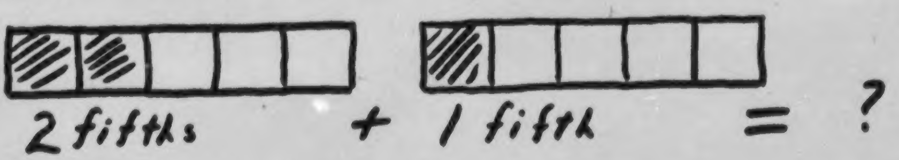
17-2 Check your students by having them add like fractions using strips of paper like those mentioned in 15-2. After one pupil holds up 2-fifths and another 1-fifth, and then they put the two together, ask: How many pieces are there together? What is the size of each piece? Is the answer fifths or is it tenths?

Give similar exercises using word names, e.g. 4 thirds + 2 thirds = ?

17-3 Make sure there is mastery of objective 15 before introducing addition of like fractions.

Use the fraction bars described in objective 15 to demonstrate addition of fractions using the oral names for the fractions. Write problems for the concrete strips using the names, e.g. 3 fourths + 1 fourth = ?

After using concrete objects and word names to teach addition, make a careful transition to diagrams for addition. While the picture does show 10 parts, each whole thing shows just 5 parts. The picture shows two whole things. Make sure the unit is well understood. Since each unit has 5 parts, the answer will be fifths — not tenths.



After mastery of addition using diagrams, use fraction symbols. If trouble with symbols, revert back to the oral word names and/or to diagrams or concrete objects.

$$\frac{2}{5} + \frac{1}{5} = ?$$

18. AR-III-B-7 (Grade 7) 56%
Add Two Mixed Numbers With Like Denominators
Items 121-125

Given two mixed numbers with like denominators, the learner will identify the sum.

$3\frac{3}{4} + 1\frac{1}{4} =$ □

F $4\frac{3}{4}$

G 5

H $5\frac{4}{8}$

J 6

18-1 Performance and sources of errors are similar to those for objective 17 but with errors also in regrouping. Item 124 (65% correct) required regrouping $4\,\frac{4}{4}$ to get 5, and item 125 (39% correct) required regrouping $5\,\frac{11}{8}$ to get $6\,\frac{3}{8}$.

The results, 56%, are lower than that for 1973. Much of this can be attributed to changes in the foils of two items. For instance, the item $2\frac{7}{8} + 3\frac{4}{8} = \square$ was given the foil $5\frac{11}{16}$ instead of 8 and the results on this item went from 52% correct in 1973 to 39% correct in 1974.

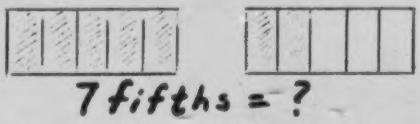
18-2 Use items from objective 17 to see if pupils can add like fractions.

Use concrete objects, diagrams and fraction symbols to get 1 as another name for $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, etc.

Check the meaning of mixed forms. For example, does $3\frac{3}{4}$ mean $3 + \frac{3}{4}$, $3 - \frac{3}{4}$, $\frac{3 + 4 + 3}{4}$, or $3 \times \frac{3}{4}$? Check such examples using oral and written forms.

See if the pupil understands the relative size of fractions in relation to 1. Say: Is there as much as a whole thing in $\frac{3}{8}$? in $\frac{5}{8}$? in $\frac{8}{8}$? in $\frac{12}{8}$?

Use concrete objects to see if a fraction such as $\frac{7}{5}$ can be seen as a whole and $\frac{2}{5}$ more.



Use the fraction strips described in objective 15 to show regrouping fractions as a whole number and a fraction.

19. AR-III-C-4 (Grade 7) 67%
Subtract Fraction From Mixed
Number/Like Denominators
Items 126-130

Given a mixed number and a fraction with like denominators, of 2, 3, 4, 6, or 8, where no regrouping is necessary, the learner will find the difference.

$2\frac{5}{8} - \frac{3}{8} = \square$

F $1\frac{2}{8}$

G $2\frac{2}{8}$

H $2\frac{3}{8}$

J $2\frac{8}{8}$

19-1 Performance and errors on these items were similar to those for objective 18.

Interpretation, further diagnosis and teaching suggestions are very similar. Examples should be included such as $6\frac{2}{3} - 1\frac{1}{3}$ as well as $6\frac{2}{3} - \frac{1}{3}$.

Note: See the diagnostic test for fractions in Section V.

20. AR-III-C-6 (Grade 7) 43%
Subtract Common Fraction From Whole Number
Items 111-115

Given a whole number and a common fraction with a denominstor of 2, 3, 4, 6, or 8, the learner will find the difference with or without the use of fractional parts.

$8 - \frac{5}{6} = \square$

A 7

B $7\frac{1}{6}$

C $8\frac{1}{6}$

D $8\frac{5}{6}$

20-1 There were two errors with high frequency: 1) decreasing the whole number by 1 and merely affixing the fraction, e.g. $9 - \frac{5}{8} = 8\frac{5}{8}$; 2) adding instead of subtracting, e.g. $9 - \frac{5}{8} = 9\frac{5}{8}$.

Results for the two years, 1973 and 1974 are identical with 43% meeting criterion and with 43% missing at least 3 of the 5 items.

20-2 Read the sentence aloud: $8 - \frac{5}{6} = \_\_$. Does it say to add or to subtract?

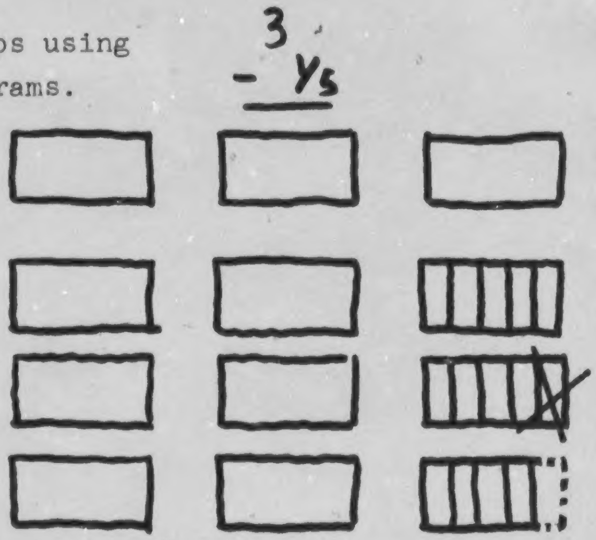
Give at least two fraction names for 1, e.g. $1 = \boxed{\frac{4}{4}}$

Name 8 as 7 and so many fourths, $8 = 7\frac{?}{4}$.

Does $3 = 2\frac{4}{4}$ or $2\frac{14}{4}$? (This checks to see if the child regroups with 10 as with whole numbers or if the correct fraction is chosen.)

See if pupils can solve such problems using the fraction bars described in objective 15.

20-3 Illustrate the subtraction in steps using the concrete strips and also using diagrams.



Begin with 3 whole things

Mark one into fifths so that $\frac{1}{5}$ can be subtracted.

Cross off the one-fifth

How much is left?

Give the students practice with oral exercises such as:

$1 - \frac{1}{2} = ?$ $1 - \frac{1}{4} = ?$ $1 - \frac{3}{4} = ?$ $1 - \frac{2}{3} = ?$

Encourage the students to quickly rename 1 and perform the subtraction.

MULTIPLICATION WITH UNIT FRACTIONS

21. AK-III-D-4 (Grade 7) 68%
Multiply 2 Unit Fractions
Items 131-135

Given two unit fractions with denominators less than seven, the learner will identify the product with or without the use of a model.

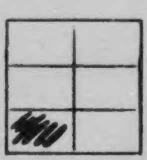
$\frac{1}{3} \times \frac{1}{2} = \square$

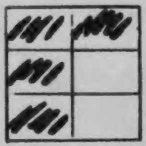
A $\frac{2}{6}$
B $\frac{1}{12}$
C $\frac{1}{6}$
D $\frac{2}{5}$

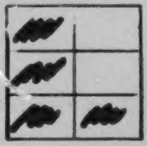
21-1 Twelve percent of the pupils got none correct and 56% got all five correct. The percents correct are probably artificially high because of the ease in merely applying the method for multiplying whole numbers, i.e. just multiply the two numbers next to each other.

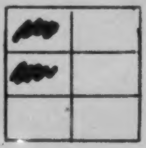
Changes in the foils of two items from 1973 resulted in lower percents correct. For instance, choice A in item number 131 was $\frac{1}{60}$ in 1973 and only 3% selected this choice. In 1974, 14% selected $\frac{2}{6}$ and 6% fewer got the item correct.

21-2 Check further to see if pupils understand multiplication of fractions using either concrete objects or diagrams. Say: Which picture shows $\frac{1}{2}$ of $\frac{1}{3}$? (ans. a) Which shows $\frac{1}{2} \times \frac{1}{3}$? (Ans. a)

a


b


c


d


See if pupils can draw diagrams. Have them draw a picture to show $\frac{1}{3}$ of $\frac{1}{4}$.

Further check their understanding of multiplication by asking:

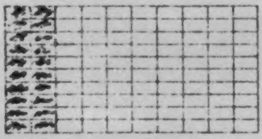
Is $\frac{1}{6} \times \frac{1}{5}$
a) greater than $\frac{1}{6}$?
b) greater than $\frac{1}{5}$?
c) less than 1?
d) greater than 1?

(Ans: c)

DECIMALS - MEANING

22. AR-IV-A-3 (Grade 7) 85%
Identify Illustrated Decimal Fraction
Items 106-110

Given a model of a fraction illustrating hundredths, the learner will identify the decimal fraction as illustrated.



F .20

G .12

H .43

J .8

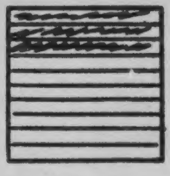
22-1 The results for this objective are not typical of the overall level of performance on decimals. The remaining questions relating to decimals should be examined for a coherent interpretation.

A lack of effective distractors resulted in an artificially high level of attainment for this objective. For example, in item number 106 the correct answer .20 was chosen by 87% of the pupils, obvious incorrect choices such as 20, 2.0 and 0.020 were not available. Results from other questions suggest very strongly that many pupils would have chosen responses such as these.

22-2 For further diagnosis, show a picture with 7 out of the 100 squares shaded. Say: Write a fraction for the part shaded ($\frac{7}{100}$). Now write a decimal for the same amount (0.07). Then repeat with 10 squares shaded and with other 2-digit numbers to the right of the ones place.

Show 99 out of 100 shaded. Say: Write the decimal (0.99). Now shade the remaining square and have pupils write the fraction ($\frac{100}{100}$) and also write the decimal (1.00).

22-3 Use a square to teach both tenths and hundredths. Begin teaching tenths by cutting the square into ten equal size slices. Have pupils write both fractions and decimals for various amounts of the square. Say: What fraction shows the part in the first row? ($\frac{1}{10}$) in 3 rows? ($\frac{3}{10}$), etc. After establishing fractions for tenths, show tenths with decimals.



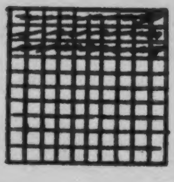
$\frac{3}{10}$ or 0.3

$$\frac{3}{10} = 0.3$$

Using the 0 to show no ones will help in teaching the relation of tenths place to tens place. Tenths is one place to the right of ones place and tens is one place to the left.

1 1 . 1
tens ones tenths

Count by tenths, 0.1, 0.2, . . . 0.9. Then add one more tenth. This gives $\frac{10}{10}$ or 1. So the next number after 0.9 is 1.0, when counting by tenths.



$\frac{3}{100}$ or 0.03

Use the same square to teach hundredths. Split each row into 10 equal-size parts. Note that there are 100 squares now in the 1 whole unit. Say: What fraction goes with 3 squares ($\frac{3}{100}$)? What decimal (0.03)? Note that hundredths is two places to the right of the ones place and that hundreds is two places to the left.

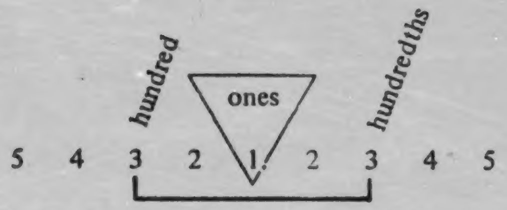
1 1 1 . 1 1
hundreds ones hundredths

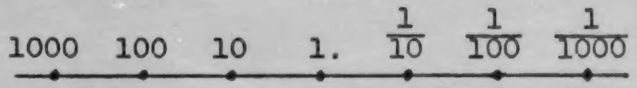
Write decimals for squares to 0.09. Then count 1 more square. Say: This is $\frac{10}{100}$. You can rename $\frac{10}{100}$ as $\frac{1}{10}$. Do you see 1 row out of ten covers the same amount as 10 squares out of 100? Since you can rename $\frac{10}{100}$ as 1 tenth and no more hundredths, you can write 0.10. You can read 0.10 as "1 tenth and 0 hundredths" or as "10 hundredths".

Continue questions such as these for decimals to 0.99, reading each decimal as tenths plus hundredths or just as hundredths. When there is difficulty in understanding, revert to fractions and to the 100 square.

The symmetry of the place values should be connected to the place value names.

a) The decimal point indicates the place of the ones. The ones place is the middle place in this symmetry.
b) The 3's are in the hundreds and hundredths places respectively from left to right.
c) Each place is $\frac{1}{10}$ as much as the place at its immediate left.





$$\frac{1}{10} \times 1000 = 100 \qquad \frac{1}{10} \times \frac{1}{100} = \frac{1}{1000}$$

To encourage the students in obtaining a feeling for the comparison between 100, 1 and .01, cut construction paper up into the following sizes: 1 meter by 1 meter square; 1 decimeter by 1 decimeter square; and 1 centimeter by 1 centimeter square. Let the decimeter square represent 1. Cut several copies of each size square and let the students represent numbers like 203.05 by counting out and arranging the squares under each numeral.

23. AR-IV-A-7 (Grade 7) 26%
Name Place Values of Decimal Fraction
Items 96-100

Given a decimal fraction of no more than three places, the learner will name the place value of each digit without aids.

.74 means which one of the following?

F 7 hundredths + 4 tenths

G 7 tenths + 4 thousandths

H 7 ones + 4 hundreds

J 7 tenths + 4 hundredths

23-1 These test items are more related to expanded notation using word names than measuring the stated objective. To test the objective, a pupil needs only name the place value (tenths, hundredths, etc.) of each digit in a numeral.

The results on this objective, more so than the items for objective 22, show that pupils do not understand decimals well. It was common for pupils to reverse the values of the place, for example to say that .74 means 7 hundredths + 4 tenths instead of 7 tenths + 4 hundredths.

23-2 For further diagnosis, ask: Is the value of 7 in 0.74 hundredths or tenths? Can you use a fraction to show what the 7 means? ($\frac{7}{10}$) In what place is the 4? (hundredths) Write the hundredths digit as a fraction ($\frac{4}{100}$)

Ask: Name 0.74 as a fraction with hundredths. ($\frac{74}{100}$) Now name the tenths ($\frac{7}{10}$) and hundredths ($\frac{4}{100}$) and show them together ($\frac{7}{10} + \frac{4}{100}$). Now show with decimals (0.7 + 0.04).

23-3 Stress the naming of decimals by reading the number to the right of the decimal point as if it were a whole number. Then give the name of the place value of the last place to the right, e.g. 0.74 means 74 hundredths.

Relate the naming of the places for decimals to the right of the decimal point to the names of the places for whole numbers, e.g. tenths and tens, hundreds and hundredths. See the teaching suggestions for objective 22.

Show how $\frac{1}{10}$ (0.1) can be named as $\frac{10}{100}$ (0.10) or as $\frac{100}{1000}$ (0.100). Show that 0.23 can be looked at two ways, as $\frac{2}{10} + \frac{3}{100}$ or as $\frac{23}{100}$. Similarly 0.23 can be seen as 0.2 + 0.03.

Find examples where decimals for tenths and hundredths are important, e.g. barometric pressure, rainfall, etc.

ADDITION AND SUBTRACTION WITH DECIMALS

24. AR-IV-B-3 (Grade 7) 71%
Addition/Subtraction Decimal Problems - Tenths
Items 136-140

Given a verbal problem involving addition and subtraction of decimal numbers involving only tenths, the learner will find the answer.

Emma bought two packages of hamburger. One weighed 1.7 pounds and the other weighed 2 pounds. How much did she buy all together?

F .3 pounds

G 1.3 pounds

H 1.9 pounds

J 3.7 pounds

24-1 Four of the five problems used only tenths and the answers did not contain choices of incorrect decimal point placement. On these four questions, the percent correct was about 80. On item number 140, using 1.7 and 2 requiring addition, only 55% got it correct with 36% choosing 1.9 as a correct answer.

The source of error is traceable to poor understanding of decimals. The incorrect choice, 1.9, should be easily eliminated by using estimation and the meaning of decimals.

The foils for item number 136 were changed from 1973 to allow the incorrect choice resulting from adding instead of subtracting. This incorrect choice was selected by 12% and the correct result was 9 percentage points lower in 1973.

24-2 Check the students on tasks like the following:
1. Which is greater 1 , 1.7 , or 2? Why?
2. Arrange in order, beginning with the least. 0.1 , 1 , 0.12 , 2.1
3. What is the whole number just before and just after 3.6? (3 and 4)
4. Is the sum, 6.2 + 5 as much as 11? Why?

24-3 Try these teaching suggestions:

1. Write decimals as mixed numbers to help in getting the idea of relative size, e.g. $3.6 = 3\frac{6}{10}$. So 3.6 is between 3 and 4.
2. Estimate the sums before finding them, e.g. 4.2 + 6 is about 10.
3. Begin with decimals such as 4.3. Count on by ones — 5.3, 6.3, etc. Show counting on by ones is the same as adding; so 4.3 + 1 = 5.3.
4. Relate adding tenths to a car or a bike odometer. If an odometer shows 23.4 and you drive 5 miles, what does it show then? (28.4)
5. Show that the ones places are lined up when adding whole numbers. For example 26 + 152 + 5 is found by arranging all the ones under each other; thus all the other places are lined up also.

$$\begin{array}{r} 26 \\ 152 \\ +\quad 5 \\ \hline \end{array}$$

Similarly, to add 0.4, 0.32 and 3, the ones should be lined up. In doing so, all the other places are then lined up also.

$$\begin{array}{l} 0.4 \\ 0.32 \\ +\ 3 \\ \hline \end{array}$$

Then zeros can be placed to the right so there will be a digit in each column. Putting in the zeros just helps one keep track of the numbers to add.

$$\begin{array}{r} 0.40 \\ 0.32 \\ +\ 3.00 \\ \hline \end{array}$$

25. AR-IV-B-6 (Grade 7) 13%
Addition/Subtraction Decimal Problems - Tenths and 100ths
Items 141-145

Given a verbal problem involving addition and subtraction of decimal numbers involving tenths and hundredths, the learner will find the answer.

Gloria ran .56 miles. Sue ran .3 miles. How much farther did Gloria run than Sue?

F **.23 miles**

G **.26 miles**

H **.53 miles**

J **.59 miles**

25-1 The lack of understanding of decimals is clearest in the results on this objective. It is obvious that the place value of decimals is not well mastered and it is from this lack of understanding that pupils do not "line up" the decimal points when they add or subtract. On item number 142, 70% selected .53 miles and only 13% selected .26 miles. Compare this with the fact that on the two items which did not require placement of the decimal, the students scored 80% and 68% correct.

Performance is very, very low when it should be relatively high. Improvement will likely come when objectives 22 and 23 are mastered. By starting work on decimals in grades 3 and 4, maintaining it in grades 5 and 6 and by increasing the amount of time given to decimals in each grade, 3 through 6, we should see major improvement.

25-2 Check the students on tasks like the following:

1. Which equals 0.8? 0.08 , 0.80 , 8.0 , 0.800
2. Where can you put more zeros and not change the value? (to right of decimal point)
3. Is $\frac{6}{10} + \frac{8}{10}$ as much as 1? Is 0.6 + 0.8 as much as 1? more than 1? Is 0.66 + .8 as much as 1? more than 1?

26. AR-IV-B-9 (Grade 7) 28%
Addition/Subtraction Decimal
Problems - Vertical
Items 66-70

Given an addition and subtraction decimal problem in horizontal or vertical form with no more than five (5) digits and no more than three (3) decimal places, the learner will find the sum or difference.

1.3 + .52 + .037 = ☐

A .102

B .687

C 1.857

D 2.19

26-1 When decimal points are already aligned in vertical form, pupils did relatively well (79% and 73% correct, respectively, on two items in vertical form). Item number 66, .231 - .12 = ☐ requiring subtracting hundredths from thousandths where pupils must do their own alignment was worked correctly by only 20% of the pupils. Forty-two percent got this item correct in 1973. Foil H on this item was changed from .231 to .219 and 69% selected this incorrect answer.

26-2 Check the students further by having them work problems like:

$$\begin{array}{r} 0.274 \\ -\ 0.162 \\ \hline \end{array} \qquad \begin{array}{r} 0.855 \\ -\ 0.321 \\ \hline \end{array}$$

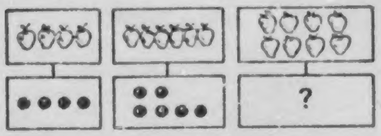
Then give them some problems in horizontal form and watch how they set the problems up in vertical form. See if they can write a number like 0.62 to show thousandths (0.620). Give them problems like: Write the second number as thousandths and subtract, $0.452 - 0.26 = \square$

26-3 For problems involving "ragged decimals", for example, $2.1 + 3 + 4.05$, teach the students to rewrite the addends before computing. Thus, $2.1 + 3 + 4.05$ becomes $2.10 + 3.00 + 4.05$. Give practice in transferring addition problems from work or equation form to vertical form with the decimal points aligned. Stress having some feeling for the reasonableness of the answer (approximation) by making sure that the students are familiar with the magnitude of the amounts under consideration. For example $8 - 1.6 \neq 1.2$ because 1.6 + (the answer) must equal 8. Since 1.6 is about 2, then the answer is about 6.

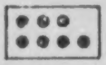
RATIOS

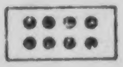
27. AR-IV-15 (Grade 7) 43%
Identify Pair of Sets/Equivalent Ratio
Items 156-160

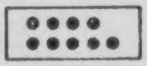
Given a picture of sets paired in (a) a one-to-one, (b) a many-to-one, or (c) a one-to-many ratio and part of another pair, the learner will identify the pair that keeps the ratio equivalent.

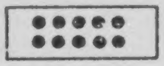


Which group of dots below completes the pattern shown above to make the ratio equivalent?


F


G


H


J

27-1 The test items do not measure understanding of equal ratios; in fact, the objective itself — while intended to do so — probably does not get at understanding of equal ratios. Consideration should be given to revision of both the objective and the test items used.

Most of the incorrect responses suggest that children are merely completing a number pattern, e.g. 1, 3, 5 rather than looking at the ratio of two numbers being the same as the ratio of two other numbers.

27-2 Test understanding of equal ratios using physical situations with which the children are familiar. For example, if 1 can of concentrated orange juice is used with 3 cans of water, how many cans of water will be used with 4 cans of concentrated juice? Write two equal ratios. ($\frac{1}{3} = \frac{4}{12}$) (So 12 cans of water would be used.)

27-3 Write the quantities to be compared so that pupils will see easily to what the numbers refer. For example,

$$\frac{\text{cans concentrate}}{\text{cans water}} \quad \text{so} \quad \frac{1}{3} = \frac{4}{12}$$

Relate finding equal ratios to finding equivalent fractions. One is multiplied by 4 so 3 is also multiplied by 4.

$$\frac{1}{3} = \frac{1 \times 4}{3 \times 4} \quad \text{so} \quad \frac{1}{3} = \frac{4}{12}$$

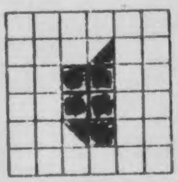
Use other practical problems such as mixing paint, batting averages (ratios), proportion of items correct on tests, etc.

AREA AND VOLUME

28. M-I-B-8 (Grade 7) 54%
Estimate Area Of A Polygon
Items 186-190

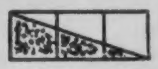
Given a polygon, the learner will estimate its area in square units.

F 3
G 4
H 6
J 7

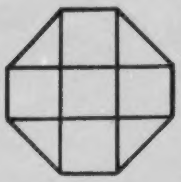


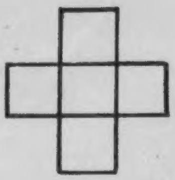
28-1 The results on this objective were poor with only 30% of the students correctly answering all five items. The most common incorrect response involved counting the number of whole units which would be required to cover the given figure. Figures with half units, such as number 188, did have a higher percentage of correct responses than those in which other fractional parts were represented.

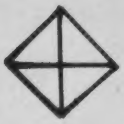
28-2 While this objective asks for an estimate, the only correct response is the exact area. Ask students whether they tried to determine the exact area or were satisfied to estimate. (An estimated answer of seven for the example may be acceptable.) Ask how many whole units and how many half units are pictured. Check to see that students do not misunderstand square units to mean squares.

28-3 Work with tiles of unit □ and $\frac{1}{2}$ unit ◺ size in covering regions will help to develop the concept of area. Geoboards and tangrams provide other motivational devices for providing concrete experiences related to areas. Students should be shown how to determine the area of triangular shaped regions as half the area of a rectangular region. i.e.  is $1\frac{1}{2}$ units.

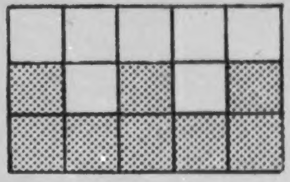
Practice finding area:

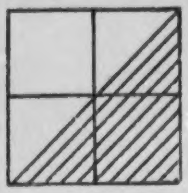


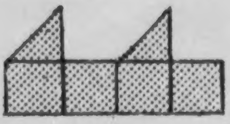




Practice finding area of shaded parts:





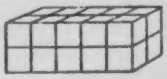


29. M-I-C-7 (Grade 7) 30%
Name Number Of Units In A Rectangular Solid
Items 191-195

Given a drawing of a rectangular solid divided into units (dimensions less than or equal to 5 units), the learner will name the number of units in it.

This a cubic unit.

A 10
B 18
C 20
D 24



29-1 Students, apparently, have not had sufficient experience in visualizing a two-dimensional drawing as a rectangular solid. That 39% of the students answered no items correctly and 16% answered only one correctly indicates that students were not simply guessing but were answering from a misconception. On four of the five items the most common response corresponded to the number of observable two-dimensional units. For example, in number 195, 48% selected 24 as their answer.

29-2 In working with students rephrase the question to: How many blocks would it take to build this figure? Determine whether the idea of a cubic unit is meaningful to them. Are they aware of the difference between square units and cubic units?

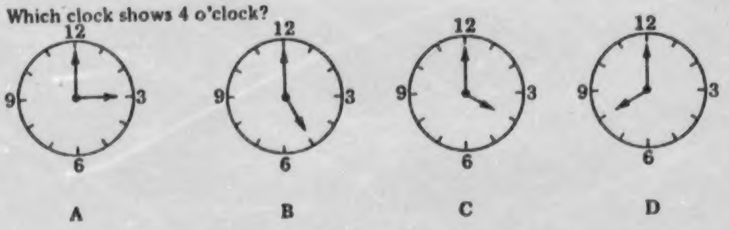
29-3 Students need practice in forming rectangular solids from unit cubes (blocks or sugar cubes) and in matching two-dimensional figures to the solids formed. Show students pictures such as used on the test and ask them to construct the figure from blocks. Provide a box with cover which can be exactly filled with unit cubes and draw the corresponding unit squares of the top and two sides. Have the students find the volume with the top on and then open the box and count cubes as a check. The art teacher may be able to help with discussions related to the drawing of such figures in art class.

TIME AND TEMPERATURE

27. M-II-A-6 (Grade 4) 94%
Telling Time
Items 106-110

Given the reading "_________ o'clock" and a clock face, the learner will identify the clock showing the appropriate time.

Which clock shows 4 o'clock?



27-1 The student performance on this item was good. On the 1974 test, the items were changed in that the numerals 3, 6, 9 and 12 were printed on the clock faces. Eighty-five percent of the students answered all 5 items correctly.

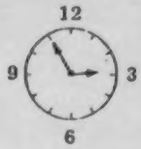
27-2 In working with students, the following points may have caused an incorrect response and should be checked. (1) A confusion between hour and minute hands. (2) A failure to mentally assign the correct numeral to the correct marking. (3) A reflection of the hands across the vertical, i.e. response D in number 109. Attempt to identify if a student's incorrect response fits any of these as a pattern.

30. M-II-A-10 (Grade 7) 61%
Tell Time
Items 161-165

Given a clock face with hands on it, the learner will choose the correct time notation.

What time does this clock show?

F 2:55
G 3:05
H 3:55
J 11:15



30-1 While the numerals, 3, 6, 9 and 12 were printed on the clock faces in this year's testing, the performance improved only slightly over last year. The results on those items showing a time greater than the half hour were lower. For example, 88% correctly identified 10:10, but only 53% were successful with 2:55.

30-2 Students may be reading the time on the sample clock as "5 minutes before three" and not being able to translate this into 2:55. Also, check to see if students are aware that until the hour hand reaches a new marking, the hour is determined by the marking the hand last passed.

30-3 Students need more practice with the use of time notation. A routine request to have students write down on their papers the time at which they complete all assignments or tests in class can provide this practice. Specific attention should be given to work on translating the commonplace verbal expressions for time, i.e. "five minutes before three" into the standard written notation "2:55".

31. M-II-A-11 (Grade 7) 71%
Identify A.M. And P.M.
Items 166-170

The learner will use A.M. and P.M. notation in writing time.

Which one of these means four hours after midnight?

F 8:00 A.M.
G 8:00 P.M.
H 4:00 A.M.
J 4:00 P.M.

31-1 The student performance on this objective was low. The range of correct responses on individual items was 74% - 82%. The most common incorrect response on each item was the one which involved an interchange of a.m. and p.m. The hour and minute notation selected was generally correct. The range for this type of incorrect response was 10% - 20%.

31-2 Ask students questions such as: Is it a.m. or p.m. when you eat breakfast? leave for school? watch a particular TV program? you see the sun set? etc. Provide a large enough set of personalized experiences that the student can associate these with the a.m. or p.m. notation. If the suggestion to have students write the time on their papers made for the previous objective is used, they could add a.m. or p.m. to the time. Ask the student with the incorrect response what a.m. (or p.m.) means to him or her. Some may think that a.m. means "after morning".

Students write their daily schedule of classes:

Grade 6	
am	pm
8:45 BELL	12:25 BELL
9:00 MATH	12:35 READ.
9:50 SCI.	1:25 SOC. ST.
10:40 MUS.	2:10 ENG.
11:30 LUNCH	3:05 DISMISS

29. M-II-C-1 (Grade 4) 50%
Identify Temperatures
Items 151-155

Given a Fahrenheit or Celsius thermometer, the learner will identify the temperature (above zero) to the nearest degree.

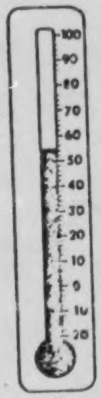
What is the temperature to the nearest degree?

A 40°

B 50°

C 55°

D 60°



29-1 One item was changed from the 1973-74 test. This was item 153 in which the picture remained the same but the response choices were changed from 0°, 55°, and 100° to those indicated. There was a drop from 88% correct to 52% correct on this item. As a result, the level of mastery on this objective dropped from 71% to 50%. Two items, where the picture showed the reading to be at a numbered scale marking, had relatively high performance levels — 87% on each.

29-2 Determine if students understand the term "to the nearest degree". Many may feel it means the nearest numbered marking. Ask students to indicate how many degrees higher the temperature is when you go from one mark to the next. Use a real thermometer and ask the student to show you the unit on the scale.

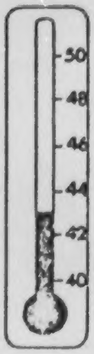
29-3 Students need practice in reading temperatures from a thermometer whether real or imitation. Construct a large cardboard thermometer with a red movable tape. Each day have a student set the temperature at the previous day's high temperature and have other students read the temperature. More general activities involving the use of scales and number lines should also be used. Have one student select a point on the scale or number line and then have another student identify the corresponding number. Have students bring in devices from home which require the reading of scales. These may include radio dials, temperature controls, electrical meters, etc.

34. M-II-C-3 (Grade 7) 84%
Read Temperatures
Items 181-185

Given a Fahrenheit or Celsius thermometer the learner will identify temperatures to the nearest degree, using the degree (°) symbol.

About what temperature does this thermometer show?

F 42°
G 43°
H 44°
J 45°



34-1 The results on four of the five items were 87% correct or better. On item 182 with a 73% correct response, 17% incorrectly responded to a temperature halfway between 42° and 44° as 45° rather than 43°.

34-2 Check to see whether the phrasing of the question, "about what temperature" may have been a problem for the student.

MONEY

28. M-II-B-3 (Grade 4) 81%
Identify Greatest or Least Amounts of Money
Items 141-145

Given three to five different amounts of money, all less or equal to $5.00, the learner will identify the greatest or the least.

Which is the *greatest* amount of money?

A **dime**
B **dollar**
C **penny**
D **quarter**

28-1 This objective was tested by three item formats: (1) Pictures of money (Items 141 and 145); (2) Word names for money, i.e. dime (Item 143; and (3) Dollar notation, i.e. $2.51 (Items 142 and 144). On item 141, 93% correctly identified the half dollar as the greatest amount of money. Since this was the largest coin pictured, this result may be artificially high. On item 145, 17% incorrectly chose the picture containing the greatest number of pieces of money (all coins) as opposed to the correct response containing only four pieces of money (2 bills and 2 coins).

28-2 Students were to identify the least amount of money in the two items tested using dollar notations and the greatest amount in those using pictures and names. These combinations make it difficult to separate the causes for incorrect responses. A clearer way of testing the understanding of the students is to ask which of the choices would buy either the most or the least amount of some article such as candy or gum. This would help identify possible confusions between value and the number of pieces of money.

28-3 This objective asks for comparisons between different amounts of money but most classroom and real life experiences ask only for counting. Try to provide experiences which require such comparisons. Classroom activities involving the operation of stores, the purchase of articles with marked prices and the determining of change are all appropriate and should fit well with the increased career awareness emphasis.

32. M-II-B-7 (Grade 7) 79%
Add/Subtract Money
Items 16-20

Given two money values, the learner will add or subtract using dollars and cents notation.

$$\begin{array}{r} \$9.27 \\ -\ \ 4.65 \\ \hline \end{array}$$

A $4.52
B $4.62
C $4.92
D $5.62

$$\begin{array}{r} \$12.53 \\ +\ \ 1.72 \\ \hline \end{array}$$

F $10.81
G $13.25
H $14.25
J $15.25

32-1 Since each item was in vertical format, the notation presented no difficulty and incorrect responses were probably the result of mistakes in addition or subtraction. The results on this objective are comparable to those on objective 4 and 5 (7th grade test) on addition and subtraction of whole numbers and on those items of objective 26 (7th grade test), addition of decimals, which were in vertical format. The only incorrect response chosen by more than 10% of the students was on item 18 where 11% subtracted rather than added as required.

33. M-II-B-9 (Grade 7) 49%
Solve Problem With Money Values
Items 146-150

Given verbal problems consisting of one or two operations involving money values less than or equal to $20, the learner will solve the problem.

Bonnie bought 3 books. Each book cost $5.75. How much change did she receive from $20.00?

F $2.75
G $2.85
H $3.75
J $4.85

33-1 While verbal problems of a one step nature should be considered minimal at this level, all five items were two-step word problems. The combination of verbal problems and two mathematical operations is too complex for many students. The writers would like to see continued effort to improve instruction involving this objective. Another year or so of investigation may show that the two-step word problems are too difficult to be considered minimal.

33-2 The items tested provide little help in diagnosis. In each case, all response choices could be obtained only through performing the correct sequence of operations and either obtaining the correct answer or making a computational mistake. Ask students to identify words they don't know. Ask them to tell you the operation that is required and to tell you what words tell them what operation to perform. Break the problems into separate small step questions and check to see how the students proceed from the small questions to solving the total problem.

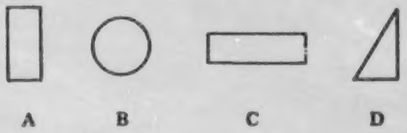
33-3 A continual emphasis on word problems is necessary throughout the mathematics program. Care should be taken, however, to insure that the reading level of the problem is not beyond the level of the student. Also, particular attention should be given to having students recognize that certain words or phrases indicate certain operations. Have the students act out the story problems with play money.

GEOMETRY

30. G-I-A-2 (Grade 4) 79%
Identify Geometric Shapes
Items 46-50

Given pictures of various shapes, the learner will identify circles, triangles, squares, and rectangles as requested.

[illegible]ch is a circle?



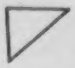
30-1 The term "figure" was dropped from the 1973 items. On this minimal objective, the percentage of correct responses in identifying each figure were: Rectangle - 73%; Triangle - 84% and 85%; Square - 89%; and Circle - 96%. The most common mistakes were to interchange the identification of rectangle and triangle. On number 49, 17% selected the triangle and on number 46, 12% selected the rectangle when the correct response was triangle.

30-2 Work on the vocabulary. Explain that the prefix "tri" has to do with three. A tricycle has three wheels — a triangle has three interior angles. Have cutouts and models available when teaching these shapes so that students can develop a feel for the shape of each figure. Also, use these terms in referring to objects in the room that are of these shapes.

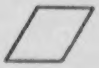
35. G-I-A-4 (Grade 7) 69%
Name Quadrilaterals
Items 171-175

Given a set of quadrilaterals, the learner will identify and name a parallelogram, a square and a rectangle.

Which figure is a rectangle?


F

G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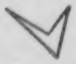
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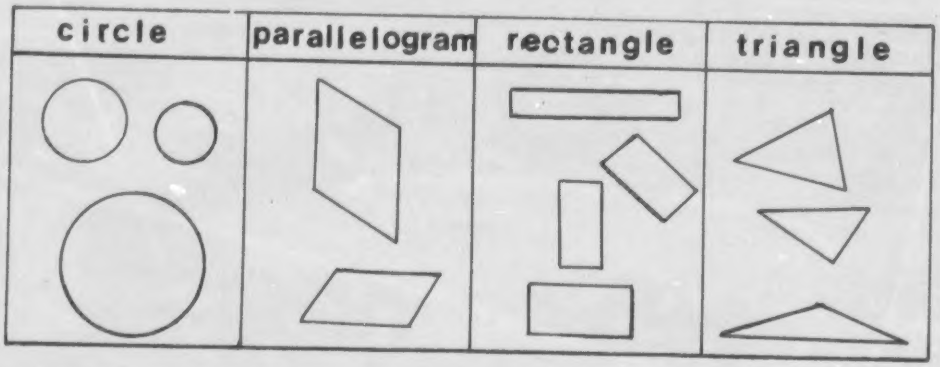
35-1 Three items were like Number 174: given the name of the figure the student is required to identify the proper figure. Two items illustrated the figure and asked the student to pick out the name. The percentages of correct response were: Pick out parallelogram - 69%; Pick out rectangle - 75%; name parallelogram - 78%; name rectangle - 81%; and pick out square - 93%.

On item 174, 75% correctly identified the rectangle and 17% selected the triangle. The corresponding percentages on objective 30, item 47, on the 4th grade test were 73% correct identification of the rectangle, and a combined 26% incorrect selection of one of two triangles offered as alternatives. In view of the results on objective 30 on the 4th grade test, the results on this minimal objective are most disappointing.

35-2 The ability to identify simple geometric shapes (other than the square and circle) and to use their names properly will benefit the secondary school mathematics student. The attainment of this objective will require an increase in the amount of time devoted to geometric topics in the elementary mathematics program.

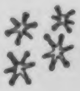
Identify and sort shapes under correct headings to be sure students can read the names.



36. G-I-B-5 (Grade 7) 79%
Identify Surfaces Representing Plane Or Part of Plane
Items 151-155

Given the description of a plane, and a part of a plane, the learner will identify surfaces which represent a plane or part of a plane.

Choose the object which suggests a plane.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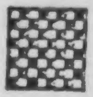
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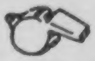
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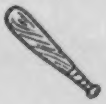
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36-1 The range of correct responses on the items measuring this objective was 81%-87%. All five items were correctly answered by 74% of the students. Those students who responded incorrectly spread their responses over all the incorrect choices. It would appear that the definition was of no help to these students and they were guessing.

36-2 Ask students who missed this objective to name each of the objects pictured to insure that they are familiar with the object and can recognize it from the picture. Ask questions like: Why do you think this is or is not like a plane? and What is a plane like?

36-3 The concept of a plane does require attention and repetition for students to attain an understanding. The classroom itself provides adequate numbers of objects which may be classified as being suitable or unsuitable representations of planes.

ALGEBRA

37. AL-2 (Grade 7) 77%
Statement Of Equality/Supply Missing Number
Items 81-85

Given a statement of equality involving addition, subtraction, or multiplication facts and containing a placeholder or letter, the learner will identify the missing number.

$n - 3 = 10$

A 5
B 7
C 11
D 13

37-1 On the three test items involving either addition or multiplication, the items were answered correctly by 86%-91% of the students. On the two involving subtraction, the percentages correct were 66% and 71%. On each item, 25% of the students chose the response which could be obtained by subtracting the numbers which appeared in the problem. For N - 3 = 10, 25% responded 7. Similar incorrect choices for the addition and multiplication items were not given. For example, 23 was not a possible choice for $N + 9 = 14$.

37-2 Questions related to these items should be directed at determining whether students computed incorrectly or whether they do not understand the sequence of operations and simple apply the indicated operation to whatever numbers appear in the problem. Have the students work a set of problems similar to the ones on the test, but not multiple choice. Analyze the answers to see the extent to which the indicated operation was incorrectly applied.

37-3 Help your students learn to read and process symbols and variables in equations. This is easily done if the words and symbols are connected with real objects and manipulations with these objects. Use the idea that the equation represents two sticks of equal length. In the equation $N - 3 = 10$, one stick is 10 units long, $N - 3$ is the length of the other stick. Ask: Does N represent a number that is greater or less than 10? Have the students show you.

38. AL-4 (Grade 7) 63%
Supply Symbol Of Equality/Inequality
Items 196-200

Given a pair of whole numbers or number phrases less than 1,000, the learner will identify the appropriate symbol of equality or inequality, > or = or <.

Which symbol should replace the □ to make 16 + 8 □ 22 true?

A > B = C <

38-1 Only 41% of the students answered all five items correctly which would seem to indicate that while students may recognize an inequality, the use of the proper symbol, > or <, to represent the inequality is not firmly established for many students. The format of the question, with the inequality included within the question, may be misleading but would not seem to account for the low performance.

38-2 Diagnosis in working with students should determine the following points: (1) Does the student recognize 16 + 8 (see sample) as one quantity or does he compare 8 □ 22? (2) Does the student recognize the quantities represented as being equal or unequal? (3) Can the student verbally express the proper relationship? (4) Can the student attach the proper symbol to the verbalization?

39. AL-5 (Grade 7) 86%
Complete Equation With One/Zero
Items 41-45

Given an equation involving one or zero, the learner will complete the sentence.

6 + □ = 6

F 10
G 6
H 1
J 0

39-1 The results on the items testing this objective were generally good with a range of 87 to 92 percent correct responses. On these items however a number of students, 5 to 10 percent, did not distinguish between the symbols for addition and multiplication (× and +). The most common incorrect response for each item was that which resulted from applying the incorrect operation.

39-2 Many of the students who respond incorrectly to these items are inadvertantly confusing the operation and/or the property of the particular identity. Have the student first write or say the operation, then write the answer. For example: 3 × 0 = □ ; first write "times", then write 0.

40. AL-7 (Grade 7) 54%
Distributive Property/Supply
Missing Value
Items 61-70

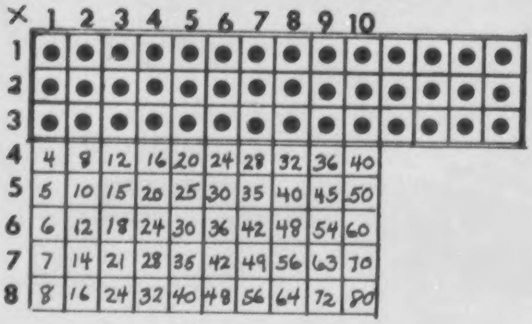
Given a numerical statement involving distributive property and a placeholder, the learner will insert the missing value.

4 × (6 + 2) = (4 × 6) + (4 × □)

A 2
B 4
C 6
D 8

40-1 While all students should have achieved a working knowledge of the distributive property by the end of the sixth grade, it is doubtful that these items represent a good test of such knowledge. Item results ranged from 59 to 72 percent correct and only 34 percent of the students correctly answered all five items. We do not believe that objective AL-7 as tested is minimal for grades 4-6.

40-2 Unless specific efforts are made to review and extend the use of the distributive property after its introduction in relation to the multiplication algorithm, i.e. 6 × 14 = (6 × 10) + (6 × 4), many students will not have reached mastery on this item. Many students get hopelessly bogged down in attempts to compute an answer from the given equation. There is little, if any, attempt to "recognize" the equation as an instance of the distributive property.



×	1	2	3	4	5	6	7	8	9	10				
1	●	●	●	●	●	●	●	●	●	●	●	●	●	●
2	●	●	●	●	●	●	●	●	●	●	●	●	●	●
3	●	●	●	●	●	●	●	●	●	●	●	●	●	●
4	4	8	12	16	20	24	28	32	36	40				
5	5	10	15	20	25	30	35	40	45	50				
6	6	12	18	24	30	36	42	48	54	60				
7	7	14	21	28	35	42	49	56	63	70				
8	8	16	24	32	40	48	56	64	72	80				

40-3 An array 3 × 14 is placed on the multiplication chart, and folded to illustrate

$$\begin{aligned} 3 \times 14 &= (3 \times 10) + (3 \times 4) \\ &= \quad 30 \quad + \quad 12 \\ &= 42 \end{aligned}$$

Shorter:

$$\begin{array}{r} 14 \\ \underline{\times\ 3} \\ \underline{12} \\ \underline{30} \\ 42 \end{array}$$

Short:

$$\begin{array}{r} 14 \\ \underline{\times\ 3} \\ 42 \end{array}$$

IV

LOCAL ACTION

The Minimum Performance Objectives for Mathematics Education in Michigan (MPOMEM) and the Michigan Educational Assessment Program (MEAP) provide a basis for identifying needs in the attainment of basic skills in mathematics in the State of Michigan. These objectives are in no way intended to prescribe a total statewide mathematics program. Since a local district's total mathematics program contains many objectives beyond those included in MPOMEM, the local district or school building is the unit in which the identification of needs and the introduction of changes can most effectively take place. The data provided by MEAP can provide a logical starting point for an organized and systematic improvement process, the consequences of which can be the improvement of a district's total mathematics program.

The statewide results, in general, indicate some small positive gain in the percentage of students attaining most objectives tested in 1974 over 1973. Based on numerous contacts statewide, the authors feel doubtful that this gain is the result of any organized local efforts to use the MEAP data during the 1973-74 school year. Rather the authors feel that the increases may be the result of such factors as improved testing procedures, simplified format and instructions on the tests, the provision of practice tests at the 4th grade level and greater acceptance of the tests on the part of teachers.

In general, the reports of specific local efforts to use MEAP data in an organized manner seem very limited. It is with these indications of limited past usage in mind and in response to the often raised question of "How can the results be used?" that this section on local action is presented.

In reading this section please keep in mind that while ideas are presented sequentially, it is not intended as a formula for using these results. It is rather a listing of hints, procedures, and methods which can be adapted to fit the needs of the individual district, building or teacher. What is feasible in a large district with central office resource personnel available may not be feasible in a small district having only two or three elementary buildings.

ESTABLISHMENT OF PRIORITY

The establishment of a priority to examine and use the results of MEAP on either a district or building basis is of prime importance, if systematic change is to take place. Work on the part of individual teachers is important and can be effective on a year-to-year and class-to-class basis at the grade levels tested, but it will not provide the continued direction needed to implement systematic program changes. The limited time of the individual teacher and the turnover of staff are such that no district or building should plan on individual teacher action as a basis for continuing use of these results. The establishment of this priority would best seem to be done on a cooperative basis between staff and administration with the understanding that two types of provisions would result from the examination. These would be (1) provision for the remediation of problems, either individual or class, identified for students taking the present test and (2) provisions for changes in mathematics program in the grades prior to and following the grade tested as such changes might be warranted by the results of the examination. Teachers would then be expected to implement the changes identified (with the district providing the materials, training, time for meetings, etc. needed to implement the changes).

INITIAL STAFF ORIENTIATION

Any use of the MEAP results should not start with the data obtained from the testing being treated in isolation. It is necessary that the test data be examined in perspective. The evidence from the two conferences on MEAP held by the Michigan Council of Teachers of Mathematics would seem to indicate that little systematic work has been done by school districts and/or the Michigan Department of Education to provide teachers with a complete understanding of how the assessment tests relate to the total set of minimal objectives and the mathematics program.

There presently exist three documents in addition to this monograph which all teachers who are concerned with the teaching of mathematics and the use of MEAP data should have available. Not only should they be available, but also a special effort should be made to insure that all teachers are familiar with the documents. These documents are:

1) Michigan Council of Teachers of Mathematics, An Introduction to the Minimum Performance Objectives for Mathematics Education in Michigan. Monograph No. 2, MCTM, February, 1973.
2) Michigan Council of Teachers of Mathematics, Michigan Educational Assessment Program: Mathematics Interpretive Report, 1973 Grade 4 and 7 Tests, Monograph No. 7, January, 1975.
3) Michigan Department of Education, Minimal Performance Objectives for Mathematics Education in Michigan, the Michigan Department of Education, 1973.

STAFF TO BE INVOLVED

Mathematics is not a group of non-related objectives to be mastered in isolation from each other. It is imperative that the assessment results be viewed from the perspective that mathematics learning is highly sequential in nature. The results of 4th grade testing have as many implications for 1st, 2nd, and

3rd grade teachers as they do for 4th grade teachers. Similarly the results of 7th grade testing have as many implications for 5th, 6th, and 8th grade teachers as they do for 7th grade teachers. As far as possible, all staff members from these grade levels should be involved in the examination of the test results and in the determination of any program changes which take place as a result of the examination. Perferably these committees would consist of representatives from all grades and would not be separate grade level committees.

NEED FOR IMMEDIATE RESULTS

With the administration of tests in the latter part of September each year and the resulting lapse of time before test results can be returned by the State, preparations should be made so as to have individual results in the hands of the 4th and 7th grade teachers as early as possible. Also, it is incumbent upon the district to see that, at the time results are returned, teachers have sufficient understanding of the objectives tested in order to immediately put these results to use. The distribution of individual student and class results to teachers should not wait for the analysis of either district or building data. Such analysis can follow the initial distribution.

It would be desirable if there were some built-in provisions in the testing process for data to be collected and retained by the teacher at the time of testing so that diagnosis and remediation might begin immediately.

While no provision for immediate feedback to the teacher presently exists, there do appear to be several ways in which a limited amount of data may be collected at the time of testing and used prior to the return of the State prepared results.

1) TEACHERS SHOULD KEEP A COPY OF THE STUDENT ASSESSMENT BOOKLET. This direction is included in the test administration manual. Every teacher who administers the test should do this. Also, copies of the test booklets should be retained by the building or district for every teacher who will make use of the test results. Since the 4th grade test covers content taught in grades 1-3 and the 7th grade test covers content taught in grades 4-6, this means retaining copies of the 4th grade test for all teachers in grades 1-4 and copies of the 7th grade test for all teachers in grades 4-7.

Even though teachers are directed to return all additional test booklets to the building coordinator, there are several ways to collect data before this is done.

2) Since vocabulary appears to be a problem on some of the test items, request students on this part of the test to circle any words in the instructions or problems which they do not understand. A quick skimming of these sections would help to point out these specific mathematical terms and phrases which present difficulty for numbers of students.

3) Another similar method would be to ask students to mark in the test booklet those sets of items which gave them difficulty. This could be done either as they take the test or after the completion of the test by having them go back through the test booklet. Obviously this would not give an exact measure of mastery but might be helpful as a starting point for diagnosis of certain difficulties.

4) Where the teacher can identify several objectives that relate directly to the content which will be taught during the month or six weeks immediately following the administration of the test, the answer sheets may be

skimmed for the results on these several objectives prior to their being submitted for scoring. If these results are copied from the answer sheets, the teacher will then in effect have some pretest results which can help in the planning of lessons over this content.

IMMEDIATELY AFTER TESTING -- THEN WHAT?

The interval between the administration of the tests and the return of the results can be utilized in a productive manner. A mathematics curriculum committee should examine the test items and the objectives tested so that by the time the results arrive back in the district, certain basic questions related to the significance of the objectives tested have been answered and the individual 4th or 7th grade teacher can focus on identifiable student deficiencies. Once this process is initiated and completed a first time, it will not represent a great task in subsequent years provided the State continues to test many or all of the same objectives each year.

The following types of questions would seem to be appropriate for examination by such a group.

1) Is the objective being tested related to the curriculum and materials being used? The developmental sequence being used and local priorities previously established could postpone the mastery of a specific objective beyond the grade level at which it is being tested. In the examination, it is important to identify the grade levels at which the objective is taught. This should include the point where the objective is first introduced, the point at which mastery is anticipated and places at which review for retention of mastery are planned.

2) Is the objective appropriate for all students at this grade level? Since no objective belongs to any fixed grade level, what must be considered is whether most students have attained the prerequisite skills needed to attain the desired objective at this grade level.

3) Are the items used to test the objective valid and of appropriate difficulty? Attempts have been made by the MCTM and Michigan Department of Education to increase both validity and appropriateness of items during the first two years of objective-referenced testing. Each year, however, as changes in items and objectives are made this point must be re-examined.

4) Is the reading level satisfactory for this grade? Students should be able to read and understand the directions if the mastery of an objective is to be measured. The answer to this question should not be confused with the need for specialized mathematical vocabulary which students should know.

5) Are the test items related to the experience of students? It is possible that the objects pictured or diagrams used in the test may not be familiar to certain groups of students. On an objective such as Objective 36, Grade 7, if students are not able to identify the object pictured, it is nearly impossible to identify which surface best represents a plane.

The answers to these questions in the form of a listing of objectives and item sets which the local district feels are appropriate to its program should be developed prior to the return of data from the State. This will enable the individual teacher to concentrate on the implications of the test results for his class without having to attempt to personally consider each of these questions.

USE OF RESULTS WITH PRESENT 4TH AND 7TH GRADE STUDENTS

The examination of the test data on either a building or classroom basis should progress on two parallel paths: (1) consider each individual objective and identify the set of pupils not attaining the objective and (2) consider the individual pupil and identify those objectives he/she has mastered and those he/she has not mastered.

The district, building and classroom summaries provided by the State give the data needed for an overview of the total program and some determination of the extent to which the objectives are being attained. The "classroom listing report" and the "individual student report" provide the necessary data for work with individual students and classes.

The classroom listing report may be scanned to obtain a quick identification of all students in a class who did not attain a given objective. A helpful idea may be to color code this report by circling in red or blue all objectives missed. This will help in the recognition of patterns of objectives mastered or missed for both individuals and for the total class.

The individual student report provides the basis for beginning some diagnosis of student difficulties. The specific incorrect response is indicated for every item a student answers incorrectly. On many of the objectives these responses can be examined for clues to misconcepts or incorrectly applied computational procedures. The teacher must be prepared to follow up on these clues and conduct a more thorough diagnosis.

Once the initial examination has taken place, the question as to whether or not the test results accurately reflect the student's knowledge should be considered. For the total group it may be that the objective has been taught as a part of the mathematics content covered since the testing took place. On an individual basis, the teacher may be aware that problems related to the testing situation, i.e., reading difficulties or emotional problems, may have caused the student to fail to reach a mastery level.

In those buildings having a learning specialist, this would be a point at which this specialist could be involved. Many of the conceptual problems this person works with may also be identifiable from this test. The learning specialist can also serve as a resource for diagnostic and remedial ideas.

In some cases, it may be appropriate to administer a teacher-made test as a check on the results obtained from MEAP.

Once it is accepted that results reflect the knowledge of the students, the organization of learning experiences can be undertaken. A limited approach to this task would seem to be appropriate to begin with unless the district is able to provide the resources necessary for a comprehensive approach. Such a limited approach would concentrate on possibly four or five objectives which were identified as being most in need of remediation either because of poor overall performance or because of their importance for the study of subsequent topics in the mathematical sequence. Success with these objectives could then serve as a model for expansion of efforts to a larger number of objectives.

A form, similar to the following sample, would serve to help in identifying the objectives and also in recording the progress and results of such an effort.

MEAP OBJECTIVES
BUILDING IMPROVEMENT EFFORT RECORD FORM

I. Number and title of objective selected ____________________

This information can be taken directly from the building summary or any of the test forms that indicate the short form of the objective label.

A. What criteria was used for selection of this objective?

1.____ Educational significance

This category would be used for any objective that was selected because of the objective's specific importance, regardless of the number of students who had not mastered it.

2.____ Distance from an arbitrary mastery level

If you determined for your class or building that X-percent of your students should have mastered this objective, and this objective was selected because less than X-percent achieved the objective this category would be checked.

3.____ Deviation from a norm

Check this item if you compared your class or building score with a district or state score and selected the objective because the class or building score was lower.

B. What students are identified for involvement?

1.____ Percent and number of students who did ~~not~~ master this item on MEAP test.

These numbers can be taken directly from the classroom or building summary.

2.____ Number of students to be involved

This is the number of students who are identified for further instruction or help on the objective identified. This may differ from B(1) because of changes in population since the administration of the test; the possible inclusion of some pupils who achieved mastery with 4 or 5 correct, or because other identification measures are used.

II. Improvement effort

A. ______ Number of teachers

The number of teachers who work with the students identified where this is a building effort.

B. ______ Number of students who have not mastered this objective on pre-testing for this effort.

This number should be based upon any pre-testing effort done. It might not require formal test evaluation. Informal diagnostic work done by the teacher could be used. The number of students will be part or all of the students identified in I-B. If no pre-testing is done this could be left blank.

C. What instructional efforts were made to assist these students to master this objective?

Identify the approaches used to instruct the students.

D. ______ What percent of the identified students mastered this objective?

When you did the final post-testing of the students you have identified to work on this objective, how many of them had attained mastery? Mastery here should follow the same model as in the MEAP format, i.e., 80 percent.

III. E. ______ Starting date of work on this objective

______ Date of post-test on this objective

In the planning of learning experiences for a class it is necessary to plan in three ways. (1) Plans must be made for all students who missed a given objective. (2) Plans must be developed for each student to work on the set of objectives he/she missed. (3) Plans must be made for all those students who mastered all of the objectives.

It must be recognized that even where only 30% of the students mastered an objective, the presentation of a total class lesson that does not provide either enrichment or alternatives for this 30% of the students would not be wise planning.

The first step in planning learning experiences for those students who have not attained mastery should be the diagnosis of specific problems. This process may take the form of either a written sequence of questions or an oral questioning of the student. If a written test is used the test should contain items designed to determine whether or not the student has attained prerequisite skills and concepts. Also this test should be open-ended rather than multiple choice so that all errors can be identified. For those items which are non-computational, or as a follow-up to a written test, an oral discussion of the items with the student can be helpful. In this situation the student can be asked to explain the thought process he/she uses in arriving at an answer.

Once diagnosis of specific student problems has taken place, the options chosen for providing learning experiences will depend upon several factors including the size of the group of students, the number of classes involved and whether or not the problem is being approached from a classroom or a total building point of view.

In those situations where it has been determined that the student has previously studied the concept, the following are suggested as some of the possible approaches that might be used:

1) Return to developmental objectives -- If diagnosis of the problems related to an objective indicates a lack of basic concepts related to the objective, a sequence of developmental objectives may need to be studied to obtain an understanding of the basic concepts. Where this is necessary the Minimal Performance Objectives for Mathematics Education in Michigan can be an excellent resource. One

particular aspect which should be stressed is the manipulation of concrete objects as a part of this process. This should not be overlooked at either the 4th or 7th grade level as a tool in working with those students for whom abstraction presents a difficulty.

2) More exposure to the topic -- In those situations where it can be identified that students have not had sufficient experience with an objective to have achieved mastery it may be appropriate to simply provide the additional experience needed.

3) Use of new materials or teaching methods -- Students are human. A change in materials or methods, provided it is mathematically and pedagogically correct, can provide a change which can help renew the interest of some students. No one method or material is correct or appropriate for all students. This last statement should not be construed as a proposal for an unlimited proliferation of materials but rather as a plea for the provision of several well planned alternatives.

4) Learning Centers -- A number of mathematics learning centers, each with activities appropriate to the mastery of a particular objective, can be established. This can be done either in the classroom or, where available, in an instructional materials center. Wherever the centers are located, one or more centers should consist of activities which would be appropriate for students who have mastered all other objectives. In this way any stigma of the learning centers being only for students who had missed objectives would be eliminated.

5) Varied student groupings -- One method of achieving varied student groupings would be to set aside a time each day during which all 4th or 7th grade students would be assigned to groups according to the objectives they had not mastered. While these methods might work most efficiently where there

is a sufficient number of staff members to work with each of the groups, this is not a prerequisite. In some situations, groups can work without direct adult supervision.

The formation of groups such as this can provide an excellent way for building principals to involve themselves in teaching situations.

6) Cross-age Tutoring -- This type of student pairing may be done in one of two ways. Older students may be paired with students who have not mastered objectives to provide them with individual assistance or students who have not achieved mastery may be paired with younger students who need help with the developmental tasks related to the specific objective. This latter approach can provide the students with the desire to master the objective in order to be able to help the younger student. Either of these pairings, however, requires close supervision and careful planning.

7) Intra-class pairing of students -- Students can be paired within class in such a way that, if the first student missed objective A and mastered objective B, he/she will be paired with a student who mastered objective A and missed objective B. This would allow students to work together without the stigma often attached to receiving help from a student at one's own grade level.

Regardless of the methods which are employed to remediate problems identified by MEAP results, a post-test should be developed and administered following the conclusion of the planned activities. Such a post-test should parallel the state test in format. The administration of the post-test should allow for a lapse of time between instruction and testing. In this way, mastery is being tested as opposed to immediate recall. Only through the administration of such a post-test can the adequacy of the learning experiences in developing mastery be measured.

USE OF THE RESULTS FOR PROGRAM EXAMINATION AT PRIOR GRADES

The use of the data as a basis for the determination of the adequacy of the program at earlier grade levels raises several immediate questions.

1) Is the local attainment level satisfactory? -- It should be recognized that any single percentage figure used for judging a satisfactory level of attainment is purely an arbitrary figure at this point in the development of MEAP. The determination of a satisfactory level of attainment should be based upon local factors combined with a knowledge of results as reported on a statewide basis.
2) Are the results comparable to previous years' results? All extremely wide discrepancies in results from year-to-year should be examined. Some, such as on objective 29-Grade 4-1974, may be accounted for by changes in the test. Others, where changes are positive, can hopefully in the long run be accounted for by changes implemented in your program based on previous such examinations.
3) Priority given the objective -- If low priority has been given a particular objective because of its place within the sequence of topics taught in the local district or because of some other local rationale, the level of attainment should be expected to be relatively low and may be accepted as satisfactory attainment.

For the 7th grade tests, districts may obtain feeder school reports which provide a building summary for groups of students on the basis of the elementary school attended in the sixth grade. This report, which must be requested by the district at the time tests are submitted for scoring, should be obtained and used by 5th and 6th grade teachers in studying their program. Since the purpose of this report is to provide building data and it is assumed that in general 4th grade students are in the same building as in the previous year, no such report is available for 4th grade testing.

The determination of an unsatisfactory level of attainment on one or more objectives leads to a need to determine possible reasons for such low levels. The areas to be examined seem to fall into the following four categories: test administration; teacher knowledge and understanding of the performance objectives; inadequate instructional materials; and inappropriate instructional approaches.

The solution of any of these problems would call for some combination of the following approaches.

1) Inservice programs -- Inservice programs can be designed to: help teachers with test administration procedures; provide teachers with an understanding of the state minimal objectives and the state assessment tests; introduce new teaching methods; introduce new materials into the program; or to provide content background which teachers may be lacking.
2) Acquisition of new materials -- It may be determined that the materials presently in use do not contain sequences necessary for the attainment of particular objectives. If this is the case, the purchase of supplementary materials or even a total change in materials may be dictated.
3) Introduction of new methods -- The introduction of different teaching approaches, math labs, learning centers, increased use of manipulative devices, etc., if only on a limited basis, may provide a starting point for improvements.
4) Changes in priority -- If the reason for low attainment on a given objective is assignment of a low priority to this objective in the total program, the assignment of a higher priority should be considered. Local conditions may still dictate the assignment of a low priority. However,

if they do, plans should provide for the attainment of the objective at some later grade levels.

5) Introduce review periods -- Where it appears that the program should have brought students to a mastery level on certain objectives much earlier than the grade at which they were tested but test results are poor, the introduction of regularly scheduled review and reinforcement periods into the mathematics program seems appropriate. The maintenance of previously mastered skills should be an integral part of every mathematics program.

The attempt to bring about change through any of these methods can be best evaluated over a period of several years. The attainment of a consistent positive change in local results is to be anticipated.

CONCLUSION

An overview of possible actions at the local level brings to the fore these few points which should be made the essence of any use of MEAP results.

1) The local district or building is the basic unit for the institution of changes.
2) An organized and systematic procedure needs to be established to make the examination and the usage of the data as meaningful as possible.
3) The progress of individuals does not conform to an artificial grade structure on sequence.
4) The objectives tested by the MEAP represent but a small part of the total set of objectives in mathematics.
5) As a beginning, the identification of four or five objectives as priority items on which improvement is needed will provide good experience. Expansion to a more full-fledged use of the data for program improvement and

increased individual attainment of objectives can be built on this experience.

5) Preparations should be make for the use of the data prior to the time at which results are returned to the district.

V

SUMMARY AND DIAGNOSTIC TESTS

The overall results are very similar to those from the 1973 testing. There is such similarity that the summary remarks on pages 46-55 of "Michigan Educational Assessment Program Mathematics Interpretive Report 1973 Grade 4 and 7 Tests" are quite applicable. The reader is referred to that book for the identification of strengths and weaknesses and for curriculum suggestions.

Four areas of arithmetic are of continuing trouble to Michigan students -- subtraction and division of whole numbers, fractions and decimals. Since these areas are of particular need, four diagnostic tests are here included.

It requires a skilled teacher to diagnose the strengths and weaknesses of his/her students. Further skill is needed in using the results to design meaningful lessons according to the needs evidenced by the results. If this booklet and these tests are helpful to teachers in doing this important job, it will have served its purpose well.

DIAGNOSES FOR INSTRUCTION - DIVISION

Teachers have a personal concern about the most effective ways to identify and overcome weak areas experienced by their own particular group of children. We know that weak areas identified in this report may not be the difficulties experienced by any one particular group of children, and that it will be necessary for teachers to use their own understanding and perspective in dealing with the needs of their own pupils.

In our search for solutions for these distinctive problems we have expanded this year's report to include sample questions for further diagnosing of instruction in particular areas, and related appropriate corrective action to be used where necessary. To ascertain any reading difficulties, children are asked to circle any words they cannot read but to try to answer the questions anyway.

The sample diagnostic test for division emphasizes four parts where weaknesses may occur:

I. Meaning of Division (Questions 1-3)

   (a) Sets must be equivalent
   (b) Writing a division equation (The numeral before the sign (dividend) indicates "what is being divided, shared, cut up, or otherwise separated")
   (c) Rewriting the division fact as a multiplication fact (inverse check)

II. Knowledge of Facts (Questions 4-5)

   (a) Missing factor for certain easy and difficult facts
   (b) Writing and answering division facts

III. Division Steps (Questions 6-11)

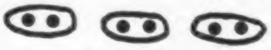
(a) DIVIDE to form as many equivalent sets as possible
(b) MULTIPLY to ascertain how many have been divided
(c) SUBTRACT to see how many remain (not divided)

IV. Division Algorithm (Questions 12-19)

(a) Knowledge of both the equation and box form
(b) Knowledge of the dividend (or product) place in the equation and box form
(c) Knowledge of:
3-place divided by 1-place, no remainder
3-place divided by 1-place, remainder
4-place divided by 1-place, remainder
5-place divided by 1-place, no remainder, zero in quotient

The test is accompanied by some suggestions for things to do with pupils who do not perform well on any one or more parts of the test.

Division - Answer Key

1.  ; 2   2. $6 \div 3 = 2$ 3. $3 \times 2 = 6$

4. a. 7, b. 6, c. 8, d. 5, e. 8, f. 6, g. 49, h. 9

5. a. $35 \div 5 = 7$ or $35 \div 7 = 5$ e. $56 \div 7 = 8$ or $56 \div 8 = 7$
b. $30 \div 5 = 6$ $30 \div 6 = 5$ f. $42 \div 7 = 6$ $42 \div 6 = 7$
c. $40 \div 5 = 8$ $40 \div 8 = 5$ g. $49 \div 7 = 7$
d. $25 \div 5 = 5$ h. $63 \div 7 = 9$ $63 \div 9 = 7$

6. 3 should be written in each of the 5 spaces

7. b. Divide — Question the child further if either of the other two answers is given--see what the thinking was

8. 15   9. a. Multiply: $5 \times 3 = 15$ 10. b. 1

11. c. Subtract: $16 - 15 = 1$

12. $\begin{array}{r} 9 \\ 5\overline{)45} \\ \underline{45} \\ 0 \end{array}$ 13. 9   14. $\begin{array}{r} 6 \\ 7\overline{)42} \\ \underline{42} \\ 0 \end{array}$ 15. 6

16. b. 91   17. a. 93   18. d. 911r2   19. c. 9350

DIAGNOSTIC TEST
DIVISION

WORKSPACE

1) Six cookies are to be divided among 3 children. Draw a picture of the cookies in your workspace, and circle each child's share.
How many cookies did each child receive?

2) Write the division equation and answer for Problem #1 on the line below.

____

3) What multiplication problem do you see in your workspace for Problem #1?
The multiplication problem is ________.

1)

4) Write the missing numbers in the boxes:

a.	$5 \times \square = 35$	e.	$7 \times \square = 56$
b.	$\square \times 5 = 30$	f.	$\square \times 7 = 42$
c.	$5 \times \square = 40$	g.	$7 \times 7 = \square$
d.	$\square \times 5 = 25$	h.	$\square \times 7 = 63$

5) Write a division fact and answer for each of the facts in Problem #4.

a.	e.
b.	f.
c.	g.
d.	h.

6) You have 16 marbles to divide among 5 children. Write the number of marbles you would give each child in the boxes shown in the WORKSPACE.

6)

☐ ☐ ☐ ☐ ☐

WORKSPACE

7) For this First Step (in Problem #6), did you

a. Multiply b. Divide c. Subtract

8) How many marbles altogether are in the boxes?

a. 5 b. 16 c. 3 d. 15

9) For this Second Step did you

a. Multiply b. Divide c. Subtract

10) How many marbles remain outside the boxes?

a. 3 b. 1 c. 16 d. 0

11) For this Third Step did you

a. Multiply b. Divide c. Subtract

---

12) Write the division problem 45 ÷ 5 = ☐ in the box form shown below.

$\overline{)\qquad\qquad}$

13) The answer to Problem #12 is _______.

14) Write the multiplication sentence ☐ × 7 = 42 in the division box form below.

$\overline{)\qquad\qquad}$

15) The answer to Problem #14 is _______.

Use your Workspace to work each of the following problems. Then circle the correct answer given below each problem.

16) 455 ÷ 5 =

a. 55 b. 91 c. 50 d. 90 e. 81

17) 465 ÷ 5 =

a. 93 b. 91r1 c. 71 d. 95 e. 75r1

18) 4557 ÷ 5 =

a. 91r2 b. 911 c. 711r2 d. 911r2

19) 46750 ÷ 5 =

a. 9530 b. 9705 c. 9350 d. 9150
e. 953r1

WORKSPACE

CORRECTIVE ACTION - DIVISION

I. Meaning of Division
and
III. Division Steps

(a) Using plastic glasses and spoons, have children illustrate the division of many even numbers by 2, say they have divided (the first step), and tell the multiplication fact they see (the second step).

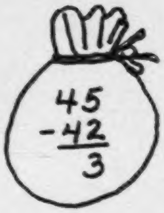
(b) Using the plastic glasses, have children illustrate the division of odd numbers by 2. This type of problem repeats the first two steps and shows the third step, subtraction, to find the remainder.

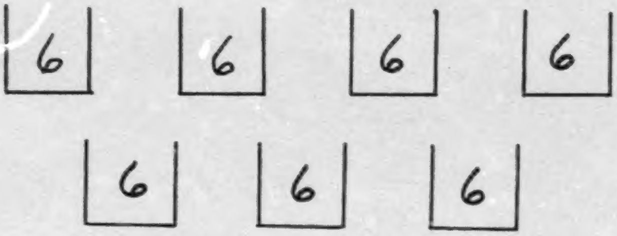
(c) Divide seven paper cookies (or real ones) among two children to introduce the idea that there need not be a remainder. Each child will get 3 whole cookies and one-half of the remaining cookie.

(d) Draw boxes and write in them the number in each equivalent set as division is performed.

$45 \div 7 = 6 \text{ r}3$

$$\begin{array}{r} 6 \\ 7\overline{)45} \\ -42 \\ \hline 3 \end{array}$$





$7 \times 6 = 42$

II. Knowledge of Facts

(a) Pair multiplication and division facts, and talk about the relationship between these operations.

(b) As the class works together, each child fills in a worksheet:

How many altogether?	How many sets?	How many in each set?	Division Fact	Multiplication Fact
45	5	9	45 ÷ 5 = 9	5 × 9 = 45
42	6			
			56 ÷ 8 =	
73	9			
0	2			

(c) ESTIMATION ONLY! Don't Overdo!

Divide each number by 9 and write the answer below it:

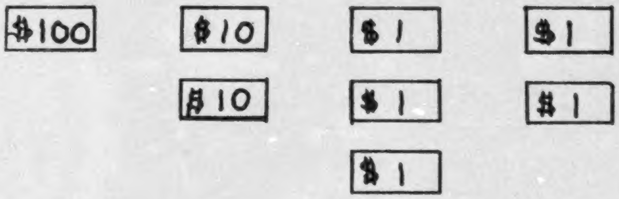
÷9	46	36	66	86	76	16	26	56	96

IV. Division Algorithm

(a) Using a hand as an automatic money-changer helps children to distribute the dividend. Both our numeration and money systems are based on TEN, and when dividing in either system we may utilize the idea of "automatic-changing" from one decimal place to another by moving a hand to the right of the desired place (behind it).

Five children will divide their rich uncle's gift of $125. How much will each child receive?

$$\begin{array}{r} \$\,25 \\ 5\overline{)\$125} \\ \underline{100} \\ 25 \\ \underline{25} \end{array}$$



Each of the children reaches for the $100 bill, but there is only one of these. (A hand behind the HUNDREDS place shows this 1 hundred.)

Someone must go to the bank to exchange it for ten $10 bills to put with the two $10 bills. (A hand behind the TENS place automatically shows the 12 ten-dollar bills.)

Learners Motto: Use your Automatic Hand-Changer for instant service.

(b) Relay Game: Divide class into five teams. Members take turns competing at the chalkboard, working a problem given by the teacher. When all children have finished, the first correct answer earns two points for that team, and all other correct answers earn one point for those teams. New members of each team take their places at the board and a new problem is given. The first team to score ten points wins the game. Children at their seats work every problem also, and are called on individually to give each answer before the judging of the board work.

DIAGNOSES FOR INSTRUCTION - SUBTRACTION

The sample diagnostic test for subtraction with whole numbers emphasizes five parts where weaknesses may occur.

I. Knowledge of Facts and the Subtraction Symbol (Questions 1-9)

II. Knowing When You Can Subtract (Questions 6-9)
The student must be aware that there is an order implied in the subtraction format and when the digit in the subtrahend is greater than the digit in the minuend, he or she must regroup.

III. Subtraction without Renaming (Questions 10-16)
Observe the student write the answers to see if he or she subtracts in the ones place first. Watch for attempts to "borrow" without thinking.

IV. Knowledge of Numeration (Questions 17-21)
- (a) Place value
- (b) Renaming

V. Subtraction with Renaming (Questions 22-30)
- (a) Watch Item Number 24 to see if the student renames when it is not necessary.
- (b) Pay special attention to Items 25, 27, 28, and 30 which involve zero.

Some suggestions for follow up.

1. Use bundles of ten sticks to illustrate subtraction. Tongue depressors or swizzle sticks are good materials. Have the student subtract by using the sticks and explain what he/she is doing while he/she records the results.

Example: $\begin{array}{r} 34 \\ -\ 18 \\ \hline \end{array}$

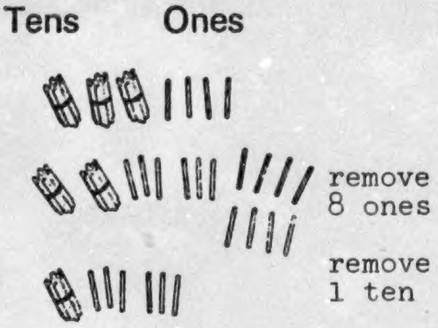
(1) Can't subtract 8 from 4, so rename 34 as 2 tens and 14 ones.

$$\begin{array}{r} 2\ 14 \\ \cancel{3}\ \cancel{4} \\ -\ 1\ 8 \\ \hline 6 \end{array}$$

(2) Now, 14 minus 8 is 6 - write the 6.

$$\begin{array}{r} 2\ 14 \\ \cancel{3}\ \cancel{4} \\ -\ 1\ 8 \\ \hline 1\ 6 \end{array}$$

(3) 2 tens minus 1 ten is 1 ten - write 1 in the tens place.



2. Have the students check their answers through addition:

a. $\begin{array}{r} 34 \\ -\ 18 \\ \hline 16 \end{array}$ so $\begin{array}{r} 18 \\ +\ 16 \\ \hline 34 \end{array}$ b. $\begin{array}{r} 504 \\ -\ 316 \\ \hline 188 \end{array}$ so $\begin{array}{r} 316 \\ +\ 188 \\ \hline 504 \end{array}$

Emphasize that subtraction and addition are inverse operations. Addition "undoes" subtraction and vice versa.

Subtraction - Answer Key

1. 11 2. 1 3. 4 4. 11 5. 3
cross out Problem 7. 6. 3 8. 5 9. 0
10. 11 11. 25 12. 30 13. 310 14. 5
15. 302 16. 2222 17. 8 18. 1
19. a. 4 tens 2 ones
 b. 6 hundreds
 4 tens
 3 ones
 c. 3 hundred 0 tens [illegible] ones
20. a. 5 hundreds
 b. 10 ones
 c. 29 tens 10 ones
 d. 700 or 7 hundreds
21. a. 6 b. 12 c [illegible]3 d. 9 e. 15 f. 12
22. 28 23. 18 24. 61 25. 170 26. 434
27. 427 28. 32 29. 204 30. 107

DIAGNOSTIC TEST

SUBTRACTION

Addition and Subtraction Problems: Look at the sign.

1. $\begin{array}{r} 3 \\ +\ 8 \\ \hline \end{array}$
2. $\begin{array}{r} 6 \\ -\ 5 \\ \hline \end{array}$
3. $\begin{array}{r} 7 \\ -\ 3 \\ \hline \end{array}$
4. $\begin{array}{r} 9 \\ +\ 2 \\ \hline \end{array}$
5. $\begin{array}{r} 8 \\ -\ 5 \\ \hline \end{array}$

Subtraction: Cross out those problems where you CANNOT subtract.

6. $\begin{array}{r} 7 \\ -\ 4 \\ \hline \end{array}$
7. $\begin{array}{r} 6 \\ -\ 9 \\ \hline \end{array}$
8. $\begin{array}{r} 5 \\ -\ 0 \\ \hline \end{array}$
9. $\begin{array}{r} 5 \\ -\ 5 \\ \hline \end{array}$

Subtraction:

10. $\begin{array}{r} 16 \\ -\ 5 \\ \hline \end{array}$
11. $\begin{array}{r} 29 \\ -\ 4 \\ \hline \end{array}$
12. $\begin{array}{r} 50 \\ -\ 20 \\ \hline \end{array}$
13. $\begin{array}{r} 426 \\ -\ 116 \\ \hline \end{array}$
14. $\begin{array}{r} 87 \\ -\ 82 \\ \hline \end{array}$
15. $\begin{array}{r} 603 \\ -\ 301 \\ \hline \end{array}$
16. $\begin{array}{r} 4738 \\ -\ 2516 \\ \hline \end{array}$

---

17. Which digit is in the tens place in $\boxed{684}$? Answer __________

18. Which digit is in the hundreds place in $\boxed{5103}$? Answer _____

19. Write each number out in its place value. Look at the example.

Example: 37 is 3 tens 7 ones

a. 42 is ____ tens ____ ones

b. 643 is ____ hundreds

____ tens

____ ones

c. 302 is ____________ ____________ ____________

20. Rename each number.

a. 50 tens is the same as _____ hundreds.

b. 500 is the same as 49 tens and _____ ones.

c. 300 is the same as 29 __________ and _____ ones.

d. 70 tens is the same as _______________.

21. Rename each number by finding the missing number.

a. 73 = _____ tens 13 ones

b. 42 = 3 tens _____ ones

c. 243 = 2 hundreds 3 tens _____ ones

d. 406 = 3 hundreds _____ tens 16 ones

e. 315 = 3 hundreds 0 tens _____ ones

f. 502 = 49 tens _____ ones

Subtraction:

22. $\begin{array}{r} 46 \\ \underline{-\ 18} \end{array}$

23. $\begin{array}{r} 53 \\ \underline{-\ 35} \end{array}$

24. $\begin{array}{r} 84 \\ \underline{-\ 23} \end{array}$

25. $\begin{array}{r} 305 \\ \underline{-\ 135} \end{array}$

26. $\begin{array}{r} 681 \\ \underline{-\ 247} \end{array}$

27. $\begin{array}{r} 700 \\ \underline{-\ 273} \end{array}$

28. $\begin{array}{r} 503 \\ \underline{-\ 471} \end{array}$

29. $\begin{array}{r} 422 \\ \underline{-\ 218} \end{array}$

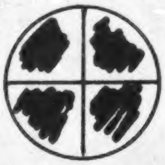
30. $\begin{array}{r} 1050 \\ \underline{-\ 943} \end{array}$

DIAGNOSTIC TEST
FRACTIONS

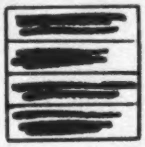
Recognizing The Unit

1. Which pictures shows one whole thing shaded?

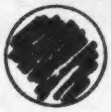
(a)



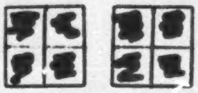
(b)



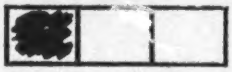
(c)



(d)

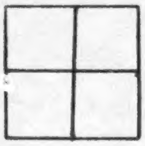


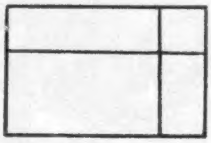
(e)

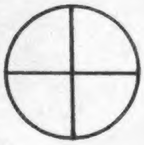


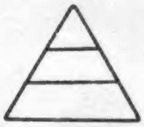
Equal-Size Pieces

2. Draw a ring if the pieces are equal size.
Put an X if they are not.

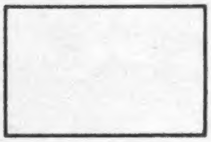


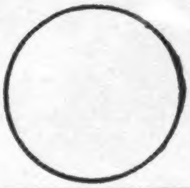


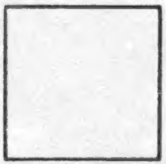




3. Mark each to show 3 equal size pieces.

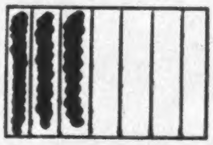


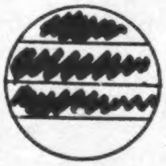


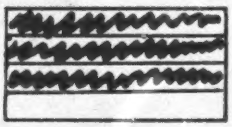


Word Names For Parts

4. Which picture is shaded to show 3 fourths?



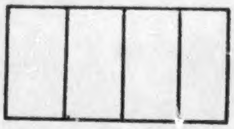




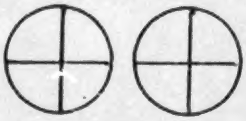
Pictures For Fractions

5. Shade the pictures to show the fractions.

$\frac{2}{4}$



$\frac{5}{4}$



6. Draw a picture to show the fractions.

$\frac{1}{2}$ $\frac{2}{5}$

Fractions And Word Names

7. Write the fraction.

2 fifths = _____ 3 sevenths = _____ 10 tenths = ______

8. Write the word names.

$\frac{5}{6}$ = _____ $\frac{2}{3}$ = ______ $\frac{1}{2}$ = ______

Addition/Subtraction--Like Denominators

9. 2 fifths
+ 3 fifths

10. 3 sixths + 3 sixths = __?__

11. $\frac{1}{3} + \frac{1}{3}$

12. $\frac{2}{10} + \frac{7}{10}$

13. $\frac{3}{5} + \frac{2}{5}$

14. $\frac{5}{10} + \frac{7}{10}$

15. $\frac{5}{6} - \frac{1}{6}$

16. $\frac{3}{10} - \frac{1}{10}$

17. $\frac{6}{3} - \frac{4}{3}$

18. $\frac{25}{100} + \frac{35}{100}$

19. $\frac{50}{100} - \frac{20}{100}$

20. Draw a picture to show $\frac{2}{4} + \frac{1}{4}$.

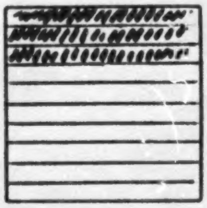
Problems

21. Four people share 1 candy bar equally. How much does each person get?

DIAGNOSTIC TEST
DECIMALS

Fractions For Tenths And Hundredths

1. Write the fraction for the shaded part of each picture.



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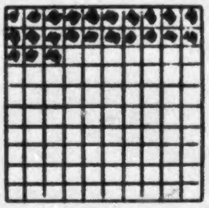

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Fractions And Decimals

2. Write the decimal for the parts shaded in Ex. 1.

____ ____

---

3. Write the decimal

$\frac{3}{10}$ = ____ $\frac{5}{10}$ = ____ $\frac{9}{10}$ = ____

$1\frac{2}{10}$ = ____ $4\frac{2}{10}$ = ____ $3\frac{1}{10}$ = ____

$\frac{4}{100}$ = ____ $\frac{12}{100}$ = ____ $\frac{47}{100}$ = ____

$3\frac{4}{100}$ = ____ $8\frac{15}{100}$ = ____ $20\frac{99}{100}$ = ____

---

4. Write the fraction

0.3 = ____ 0.9 = ____ 0.1 = ____

0.02 = ____ 0.13 = ____ 0.99 = ____

0.10 = ____ 0.40 = ____ 0.90 = ____

Tenths And Hundredths

5. Write as tenths.

$\frac{30}{100}$ = ____ $\frac{60}{100}$ = ____ $\frac{100}{100}$ = ____

---

6. Write as hundredths.

$\frac{2}{10}$ = ____ $\frac{1}{10}$ = ____ $\frac{6}{10}$ = ____

0.2 = ____ 0.9 = ____ 0.1 = ____

Addition/Subtraction

7. Add

0.2 + 0.9

$$\begin{array}{r} 0.5 \\ \underline{+\ 0.8} \end{array} \qquad \begin{array}{r} 6.9 \\ 7.3 \\ \underline{+\ 8.6} \end{array}$$

$$\begin{array}{r} 0.18 \\ 0.46 \\ \underline{+\ 0.94} \end{array}$$

0.42 + 0.69 + 0.11

---

8. Subtract

$$\begin{array}{r} 0.9 \\ \underline{-\ 0.4} \end{array}$$

0.8 - 0.3

6.8 - 1.9

---

9. Add

0.2 + 6.3

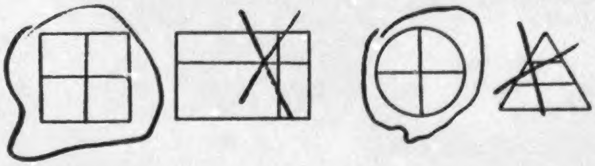
$$\begin{array}{r} 0.8 \\ \underline{+\ 0.95} \end{array}$$

6.94 + 3.2

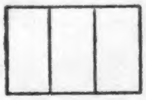
Fraction - Answer Key

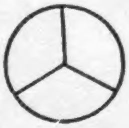
1. a, b, and c

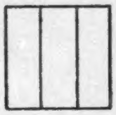
2.



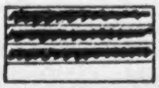
3.



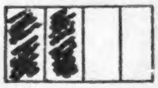


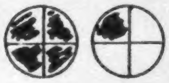


4.



5.





6. Drawings will vary.

7. $\frac{2}{5}$ $\frac{3}{7}$ $\frac{10}{10}$ 8. Five-sixths two-thirds one-half

9. 5 fifths   10. 6 sixths (or one)   11. $\frac{2}{3}$ 12. $\frac{9}{10}$

13. $\frac{5}{5}$ (or one)   14. $\frac{12}{10}$ 15. $\frac{4}{6}$ 16. $\frac{2}{10}$ 17. $\frac{2}{3}$ 18. $\frac{60}{100}$

19. $\frac{30}{100}$ 20. Pictures will vary   21. one-fourth

Decimal - Answer Key

1. $\frac{3}{10}$ $\frac{23}{100}$ 2. 0.3 0.23   3. 0.3 0.5 0.9
1.2 4.2 3.1
0.04 0.12 0.47
3.04 8.15 20.99

4. $\frac{3}{10}$ $\frac{9}{10}$ $\frac{1}{10}$ 5. $\frac{3}{10}$ or 0.3 $\frac{6}{10}$ or 0.6 $\frac{10}{10}$ or 1.0
$\frac{2}{100}$ $\frac{13}{100}$ $\frac{99}{100}$
$\frac{10}{100}$(or $\frac{1}{10}$) $\frac{40}{100}$(or $\frac{4}{10}$) $\frac{90}{100}$(or $\frac{9}{10}$)

6. $\frac{20}{100}$ or 0.20 $\frac{10}{100}$ or 0.10 $\frac{60}{100}$ or 0.60   7. 1.1 1.3 22.8
1.58 1.22
0.20 0.90 0.10

8. 0.5 0.5 4.9   9. 6.5, 1.75, 10.14

APPENDIX A

STATEWIDE RESULTS

SUMMARY

STATEWIDE SUMMARY, GRADE FOUR

MICHIGAN EDUCATIONAL ASSESSMENT PROGRAM 1974-75 (YEAR 6)

DISTRICT:
CODE:

SCHOOL:
CODE:

GRADE: FOUR

DATE TESTED: 10/74

CLASSROOM, SCHOOL
OR DISTRICT
SUMMARY

WORD RELATIONSHIPS (OPTIONAL)		
STAND. SCORE INTERVAL	FREQUENCY	PERCENT IN INTERVAL
70-74	807	1
65-69	4307	7
60-64	6514	11
55-59	7443	12
50-54	8395	14
45-49	11628	19
40-44	8482	14
35-39	6731	11
30-34	4383	7
25-29	1512	2
20-24	312	1
15-19	43	1
60557	NUMBER OF PUPILS	

PERCENT OF STUDENTS ANSWERING										
0 ITEMS CORRECT	1 ITEM CORRECT	2 ITEMS CORRECT	3 ITEMS CORRECT	4 ITEMS CORRECT	5 ITEMS CORRECT	% PUPILS ATTAINING OBJECTIVE	NUMBER OF PUPILS	OBJ. NO.	OBJECTIVE	OBJECTIVE CODE
									****** MATHEMATICS ******	
2	3	2	2	6	86	92	149472	1	INDICATE OBJECTS THAT ARE SAME SIZE	AR-I-A-6
1	1	1	1	4	92	96	149502	2	INDICATE SIMILAR GEOMETRIC SHAPES	AR-I-A-8
8	8	6	4	7	68	75	148944	3	INDICATE OBJECTS ARRANGED FULL TO EMPTY	AR-I-A-16
1	2	2	3	6	86	92	148461	4	INDICATE LONGEST AND SHORTEST OBJECTS	AR-I-A-24
2	2	3	1	3	88	91	148532	5	INDICATE FIRST AND LAST	[illegible]
2	7	4	3	9	74	83	148658	6	CHOOSE EQUIVALENT SETS	AR-I-B-7
15	6	3	2	4	69	73	148820	7	CHOOSE SETS HAVING FEWER NUMBERS	[illegible]
2	6	5	8	16	62	78	147919	8	INDICATE APPROPRIATE NUMERAL FOR POINT ON A LINE	AR-I-B-40
1	2	5	4	9	81	90	145628	9	CHOOSE GREATEST AND LEAST NUMBER	[illegible]
1	3	2	2	4	88	92	149187	10	CHOOSE NUMBER BETWEEN TWO NUMBERS	AR-I-B-44
2	3	4	9	13	69	82	149127	11	CHOOSE NUMBER BEFORE OR AFTER A NUMBER	[illegible]
2	3	4	5	18	68	86	149043	12	IDENTIFY A NUMERAL LESS THAN A 100	AR-I-B-64
2	4	10	13	15	56	71	149252	13	INDICATE NUMBER BEFORE OR AFTER NUMBER WITHIN A DECADE	[illegible]
0	1	2	5	13	79	92	145615	14	INDICATE WHICH OF 2 NUMBERS IS GREATER OR LESS	AR-I-B-67
2	4	4	5	13	71	84	147286	15	INDICATE THE VALUES OF A SET OF DIMES AND PENNIES	[illegible]
7	9	9	9	20	45	65	149326	16	CHOOSE LIST OF NUMBERS IN ASCENDING ORDER	AR-I-B-81

0 ITEMS CORRECT	1 ITEM CORRECT	2 ITEMS CORRECT	3 ITEMS CORRECT	4 ITEMS CORRECT	5 ITEMS CORRECT	% PUPILS ATTAINING OBJECTIVE	NUMBER OF PUPILS	OBJ. NO.	OBJECTIVE	OBJECTIVE CODE
0	1	3	6	16	73	89	149553	17	INDICATE GREATER OR LESS/ SCRAMBLED POSITIONS	[illegible]
2	5	5	9	22	58	80	149557	18	INDICATE NEXT NUMBER IN A SEQUENCE	AR-I-B-84
7	10	7	5	7	64	71	149413	19	INDICATE A NUMBER THAT IS A MULTIPLE OF 3	[illegible]
3	11	24	9	16	36	52	148315	20	SELECT SET WITH TWICE AS MANY MEMBERS AS ANOTHER	AR-I-B-86
0	1	1	2	13	83	96	149678	21	ADD TWO-DIGIT AND ONE-DIGIT NUMBER/NO CARRYING	AR-II-A-10
18	6	4	5	10	57	67	147743	22	NUMBER SENTENCES / SUBTRACTION	AR-II-B-9
1	2	4	9	24	60	84	148088	23	NUMBER SENTENCES / ADDITION OR SUBTRACTION - IDENTIFY OPER.	[illegible]
9	6	5	7	22	51	73	148111	24	NUMERICAL SET COMPARISONS	AR-II-B-13
1	2	2	4	11	81	92	149666	25	SUBTRACT ONE-DIGIT FROM TWO-DIGIT NUMBER / NO BORROWING	[illegible]
2	4	4	5	17	69	86	149645	26	SUBTRACT TWO-DIGIT FROM TWO-DIGIT NUMBER / NO BORROWING	AR-II-B-16
[illegible]	2	2	2	9	85	94	146613	27	TELLING TIME	[illegible]
1	3	5	9	23	58	81	147574	28	IDENTIFY GREATEST OR LEAST AMOUNTS OF MONEY	M-II-B-3
2	7	17	24	24	26	50	146935	29	IDENTIFY TEMPERATURES	[illegible]
1	3	5	8	17	62	79	149524	30	IDENTIFY GEOMETRIC SHAPES	G-I-A-2
									******** READING *********	
4	8	10	11	20	47	67	146940	1	MATCH WORDS WITH DEFINITIONS	2.1
7	8	7	7	12	59	72	146750	2	INDICATE PHRASES WITH SAME MEANING	[illegible]
5	9	8	9	13	57	70	147732	3	CHOOSE WORD APPROPRIATE TO BLANK SPACES (CLOZE PROCEDURE)	3.2
2	5	5	6	16	67	[illegible]	[illegible]	4	IDENTIFY METHOD OF ARRANGING DATA	[illegible]
3	8	12	15	20	41	61	147131	5	ALPHABETIZE WORDS THROUGH FIRST 3 LETTERS	4.4
2	4	4	5	14	70	84	149102	6	INDICATE FACTUAL SELECTIONS	[illegible]
7	7	5	3	7	70	77	148745	7	INDICATE FICTIONAL SELECTIONS	5.2
4	8	10	12	20	47	67	147269	8	INDICATE AUTHOR'S PURPOSE	[illegible]
3	9	13	17	28	29	57	146522	9	INDICATE TITLE MOST APPROPRIATE FOR SELECTION	7.1
3	8	15	21	30	23	53	147569	10	INDICATE PICTURES BEST DESCRIBING [illegible]	[illegible]
6	15	19	21	22	18	40	146482	11	CHOOSE BEST SUMMARY OF A SELECTION	7.3
5	11	15	19	25	25	[illegible]	147134	12	MATCH QUOTATION FROM STORY WITH SPEAKER	[illegible]
4	8	10	12	19	48	67	147025	13	CHOOSE ANSWER BEST DESCRIBING HOW CHARACTER FEELS IN STORY	10.3
[illegible]	[illegible]	[illegible]	15	23	32	55	149512	14	CHOOSE PHRASE BEST DESCRIBING WORD IN FIGURATIVE [illegible]	[illegible]
4	8	10	11	18	48	66	146370	15	MATCH CAUSES WITH EFFECTS	11.1
[illegible]	[illegible]	[illegible]	[illegible]	23	30	53	[illegible]	16	CHOOSE MOST APPROPRIATE [illegible]	[illegible]
4	9	13	20	26	28	54	148070	17	ANSWER LOCATIONAL QUESTION ABOUT REFERENCE SOURCES	13.1
[illegible]	[illegible]	[illegible]	9	30	41	71	147912	18	ANSWER LOCATIONAL QUESTION [illegible]	[illegible]
4	9	13	17	26	30	56	147068	19	SELECT MEANINGS, GENERALIZATIONS, CONCLUSIONS NOT EXPRESSED	14.1 - 14.3

STATEWIDE SUMMARY, GRADE SEVEN

MICHIGAN EDUCATIONAL ASSESSMENT PROGRAM 1974-75 (YEAR 6)

DISTRICT CODE:

SCHOOL CODE:

GRADE: SEVEN

DATE TESTED: 10/74

CLASSROOM, SCHOOL OR DISTRICT SUMMARY

WORD RELATIONSHIPS (OPTIONAL)		
STAND. SCORE INTERVAL	FREQUENCY	PERCENT IN INTERVAL
70-74	131	1
65-69	3260	5
60-64	6711	10
55-59	12842	20
50-54	10552	16
45-49	9777	15
40-44	11106	17
35-39	5967	9
30-34	4224	6
25-29	550	1
20-24	222	1
15-19	56	1
65798	NUMBER OF PUPILS	

PERCENT OF STUDENTS ANSWERING										
0 ITEMS CORRECT	1 ITEM CORRECT	2 ITEMS CORRECT	3 ITEMS CORRECT	4 ITEMS CORRECT	5 ITEMS CORRECT	% PUPILS ATTAINING OBJECTIVE	NUMBER OF PUPILS	OBJ NO.	OBJECTIVE	OBJECTIVE CODE
									****** MATHEMATICS ******	
3	4	6	11	23	54	77	161574	1	IDENTIFY NUMBER 100/1000 LARGER	AR-I-B-07
1	1	1	2	9	86	95	161553	2	IDENTIFY ARABIC NUMERAL	AR-I-B-09
0	1	2	3	13	81	94	161527	3	ADD 3- AND 1-, 2-, AND 3- DIGIT NUMBERS	AR-II-A-22
1	2	3	6	21	66	87	161458	4	ADD TWO OR THREE NUMBERS	AR-II-A-26
3	3	4	8	23	59	82	161565	5	SUBTRACT 2- OR 3- DIGIT NUMBER FROM 3- DIGIT NUMBER	AR-II-B-20
2	3	3	6	14	73	87	161474	6	REPRESENT REPEATED ADDITION AS MULTIPLICATION	AR-II-C-6
1	2	2	1	7	87	94	161457	7	INVERSE MULTIPLICATION	AR-II-C-11
1	2	3	4	16	75	91	161319	8	MULTIPLY 1-DIGIT NUMBER AND MULTIPLE OF 10/100	AR-II-C-13
1	3	4	5	15	72	87	161374	9	MULTIPLY 2-DIGIT AND 1-DIGIT NUMBERS	AR-II-C-15
1	3	3	5	14	73	87	161273	10	SUPPLY MISSING FACTOR / MULTIPLICATION	AR-II-D-5
5	6	4	3	5	78	83	160774	11	REWRITE DIVISION FACT AS MULTIPLICATION FACT	AR-II-D-7
3	6	7	7	13	63	76	161033	12	DIVISION / ONE-DIGIT DIVISOR, DIVIDEND LESS THAN 100	AR-II-D-9
6	10	9	8	13	52	65	160954	13	DIVISION / ONE-DIGIT DIVISOR, FOUR-DIGIT DIVIDEND	AR-II-D-15
10	9	6	4	5	64	65	158591	14	IDENTIFY CONGRUENT PARTS	AR-III-A-1
15	7	4	4	12	58	70	160724	15	IDENTIFY SHADED AREA OF FIGURE WITH FRACTION	AR-III-A-18
11	13	15	12	16	33	49	160480	16	ORDER FRACTIONS WITH LIKE DENOMINATORS	AR-III-A-19
15	8	6	5	10	56	66	160240	17	IDENTIFY SUM OF TWO FRACTIONS WITH LIKE DENOMINATORS	AR-III-B-4
4	11	11	18	24	32	56	160111	18	ADD TWO MIXED NUMBERS WITH LIKE DENOMINATORS	AR-III-B-7
5	8	8	12	17	50	67	160004	19	SUBTRACT FRACTION FROM MIXED NUMBER / LIKE DENOMINATORS	AR-III-C-4

0 ITEMS CORRECT	1 ITEM CORRECT	2 ITEMS CORRECT	3 ITEMS CORRECT	4 ITEMS CORRECT	5 ITEMS CORRECT	% PUPILS ATTAINING OBJECTIVE	NUMBER OF PUPILS	OBJ NO.	OBJECTIVE	OBJECTIVE CODE
15	15	13	14	10	33	43	160256	20	SUBTRACT COMMON FRACTION FROM WHOLE NUMBER	AR-III-C-6
12	8	6	5	12	56	68	159931	21	MULTIPLY 2 UNIT FRACTIONS	AR-III-D-4
4	5	4	2	10	75	85	160392	22	IDENTIFY ILLUSTRATED DECIMAL FRACTION	AR-IV-A-3
41	16	10	7	11	15	26	160607	23	NAME PLACE VALUES OF DECIMAL FRACTION	AR-IV-A-7
2	7	9	11	28	43	71	159822	24	ADDITION / SUBTRACTION DECIMAL PROBLEMS TENTHS	AR-IV-B-3
7	21	36	24	9	4	13	159640	25	ADDITION / SUBTRACTION DECIMAL PROBLEMS TENTHS AND 100THS	AR-IV-B-6
5	12	22	33	17	11	28	161116	26	ADDITION / SUBTRACTION DECIMAL PROBLEMS VERTICAL	AR-IV-B-9
3	9	16	28	21	22	43	159164	27	IDENTIFY PAIR OF SETS / EQUIVALENT RATIO	AR-VI-15
9	15	9	13	24	30	54	158269	28	ESTIMATE AREA OF A POLYGON	M-I-B-8
39	16	9	6	8	22	30	158093	29	NAME NUMBER OF UNITS IN A RECTANGULAR SOLID	M-I-C-7
2	6	13	18	22	39	61	159001	30	TELL TIME	M-II-A-10
4	6	9	11	21	50	71	158816	31	IDENTIFY A.M. AND P.M.	M-II-A-11
1	3	6	11	27	52	79	161541	32	ADD / SUBTRACT MONEY	M-II-B-7
7	14	15	15	20	29	49	159452	33	SOLVE PROBLEM WITH MONEY VALUES	M-II-B-9
1	3	4	8	25	59	84	158474	34	READ TEMPERATURES	M-II-C-3
2	7	9	13	18	51	69	158715	35	NAME QUADRILATERALS	G-I-A-4
4	6	6	5	5	74	79	159324	36	IDENTIFY SURFACES REPRESENTING PLANE OR PART OF PLANE	G-I-B-5
2	3	4	14	29	48	77	160853	37	STATEMENT OF EQUALITY / SUPPLY MISSING NUMBER	AL-2
4	12	10	10	22	41	63	157708	38	SUPPLY SYMBOL OF EQUALITY / INEQUALITY	AL-4
1	1	5	8	17	69	86	161426	39	COMPLETE EQUATION WITH ONE / ZERO	AL-5
9	13	12	13	20	34	54	161180	40	DISTRIBUTIVE PROPERTY / SUPPLY MISSING VALUE	AL-7
									******** READING *********	
8	12	12	13	22	33	55	158140	1	MATCH WORDS WITH DEFINITIONS	18.1
5	8	8	10	18	51	69	155034	2	INDICATE PHRASES WITH SAME MEANINGS	18.2
4	7	6	9	19	54	73	160225	3	IDENTIFY METHOD OF ARRANGING DATA	20.1
2	5	5	8	16	63	79	160587	4	ALPHABETIZE WORDS THROUGH FIRST THREE LETTERS	21.1
4	6	5	5	11	69	80	160309	5	INDICATE FACTUAL SELECTIONS	22.1
6	8	6	5	10	65	75	159810	6	INDICATE FICTIONAL SELECTIONS	22.2
4	9	12	15	22	38	60	158668	7	INDICATE AUTHOR'S PURPOSE	23.1 - 23.3
3	9	11	13	21	43	64	158657	8	INDICATE TITLE MOST APPROPRIATE FOR SELECTION	24.1
2	6	8	12	29	43	72	159142	9	INDICATE PICTURES BEST DESCRIBING MAIN IDEA IN SELECTIONS	24.2
4	8	10	13	22	42	64	158843	10	CHOOSE BEST SUMMARY OF A SELECTION	24.3
6	12	12	14	24	32	56	158421	11	CHOOSE SELECTIONS ALIKE IN IDEAS EXPRESSED	24.4
5	10	11	12	24	39	63	158261	12	MATCH QUOTATION FROM STORY WITH SPEAKER	25.3
4	11	15	18	22	30	52	158028	13	ANSWER QUESTIONS RELATING TO SEQUENCE IN STORIES	26.1
4	9	12	15	22	37	59	157866	14	CHOOSE ANSWER BEST DESCRIBING HOW CHARACTER FEELS IN STORY	27.2
3	8	11	13	21	46	67	158881	15	ANSWER QUESTIONS RELATING TO MOTIVATION	28.1
4	8	12	15	21	40	61	158816	16	MATCH CAUSES WITH EFFECTS	29.1
4	10	11	13	22	40	62	158681	17	SELECT MEANINGS, GENERALIZATIONS, CONCLUSIONS NOT EXPRESSED	29.2
2	7	8	10	23	49	72	160087	18	ANSWER LOCATIONAL QUESTION ABOUT REFERENCE SOURCES	31.1
2	5	8	23	26	35	61	160356	19	ANSWER LOCATIONAL QUESTION ABOUT NEWSPAPERS	32.1
5	9	9	13	28	36	64	155573	20	CHOOSE MOST APPROPRIATE CONCLUSION FOR A STORY	32.2

APPENDIX B

CHANGES IN MEAP MATHEMATICS ITEMS FROM 1973-74 TO 1974-75

GRADE 4 TEST

1973 Test Item Number	1973 Objective Number	Change	1974 Objective Number
1-5	23	Dropped	X
16-20	35	Dropped	X
Before 21	22	Add "Add:"	21
Before 26	27	Add "Subtract:"	25
81-85	16	Drop: put in sample section (item 84 changed)	X
86-90	34	Drop the word "figure"	30
111-115	31	Add the numbers 3, 6, 9, 12 to each clock in each item	27
129	6	Change foil letter C to letter H	6
141-145	26	Add the words "than in the smaller group?" to each question	24
148	24	Make X's more precise	22
151-155	29	Dropped; used in sample section	X
156-160	30	Dropped; used in sample section	X
168	33	Change foil A to 40°; B to 50°; D to 60°	29
180	32	Make dollar pictures larger	28

GRADE 7 TEST

1973 Test Item Number	1973 Objective Number	Change	1974 Objective Number
31-35	34	Dropped	X
41-45	7	Change each question to: "Which one of these names the same number as...."	7
59	8	Change foil B to 633; foil D to 300	8
77	12	Change problem to $80 \div 4 = \underline{\qquad}$	12
90	40	Change problem to $7 = n - 5$	37
91	26	Change foil C to .219	26
92	26	Change foil G to .687	26
96-100	22	Darken all shading	22
99	22	Foil A: .09; foil B: .19; foil C: .90; foil D: .91	22
101-105	23	Replace each comma to a + sign	23
106-110	16	Change wording in question to: "Which group of fractions below is in order of increasing value?"	16
107	16	Change foil D to $\frac{2}{6}, \frac{4}{6}, \frac{5}{6}, \frac{6}{6}, \frac{8}{6}$	16
111	15	Change foil D to $\frac{6}{5}$	15
112	15	Change foil J to $\frac{3}{2}$	15
113	15	Change foil A to $\frac{2}{4}$	15
114	15	Change foil J to $\frac{3}{8}$	15
115	15	Change foil D to $\frac{2}{5}$	15

1973 Test Item Number	1973 Objective Number	Change	1974 Objective Number
128	18	Change foil G to $6\frac{2}{6}$	18
130	18	Change foil H to $5\frac{3}{8}$; change foil J to $5\frac{11}{16}$	18
136	21	Change foil F to $\frac{2}{6}$	21
139	21	Change foil A to $\frac{2}{7}$	21
141	24	Change foil B to .6 miles; foil C to .9 miles; foil D to 1.2 miles	24
156-160	44	Dropped	X
166-170	28	Dropped	X
171-175	36	Make all thermometers larger	34
176-180	27	Change all questions to "Which group of circles below completes the pattern shown above to make the ratio equivalent?"	27
181-185	31	Add the numbers 3, 6, 9, 12 to each clock for each item	30
201-225	38	Dropped	X

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